ASTRONOMY QUALIFYING EXAM
January 2026

Notes and Instructions

• There are 5 problems. 4 of the 5 questions count as your grade on the exam. You may choose to answer only 4 questions, or the question with the lowest grade will be dropped. The other 4 questions will be used to grade the exam.
• Write on only one side of the paper for your solutions.
• Write your alias on every page of your solutions.
• Number each page of your solutions with the problem number and page number (e.g. Problem 3, p. 2/4 is the second of four pages for the solution to problem 3.)
• You must show your work to receive full credit.

Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_\oplus = 6 \times 10^{27}$ g
$M_{bol,\odot} = 4.74$ mag
$R_\odot = 7 \times 10^{10}$ cm
$T_{\mathrm{eff},\odot} = 5777$ K
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
1 radian = 206265 arcsec
a = 7.56 $\times 10^{-15}$ erg cm^{-3} K^{-4}
c = 3 $\times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
k = 1.38 $\times 10^{-16}$ erg K^{-1} = 8.6173 $\times 10^{-5}$ eV K^{-1}
e = 4.8 $\times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
G = 6.67 $\times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s = 4.1357 $\times 10^{-15}$ eV s
1 amu = 1.66053886 $\times 10^{-24}$ g

PROBLEM 1

For an artificial star with mass M, radius R, and central mass density ρ_c, assume that the density changes as a function of radial distance r from the center as

$$\rho(r) = \rho_c \left(1 - \frac{r}{R}\right).$$

(a) (2 points) Derive an equation for the enclosed mass as a function of r, i.e., $m(r)$.

(b) (2 points) Derive the relation for the total mass in terms of R and ρ_c, i.e., $M(R, \rho_c)$. What is the average density of the star, in terms of ρ_c?

(c) (2 points) Assuming the star is in hydrostatic equilibrium, write an expression for the pressure gradient, dP/dr in terms of ρ_c, R, and r.

(d) (2 points) The central pressure of this star is

$$P_c = \frac{5}{4\pi}\frac{GM^2}{R^4}.$$

Assuming an ideal gas equation of state, derive an expression for the central temperature, T_c in terms of M, R, the hydrogen mass m_{H}, and mean molecular weight μ.

(e) (2 points) If this star has $M = M_\odot$ and $R = R_\odot$ and is composed of 100% hydrogen, calculate the central temperature in Kelvins. Based on this temperature alone, what evolutionary phase do you expect this star to be in, and why?

PROBLEM 2

(a) (2 points) Describe the Fundamental Plane, including the parameters involved and how they are correlated.

(b) (2 points) Describe the Tully-Fisher Relation and how it is used in astronomy.

(c) (4 points) Assuming stars are in circular orbits, show that galaxies with similar mass-to-light ratios and average surface brightness follow the Tully-Fisher relation.

(d) (2 points) What is the reason behind the development of the baryonic Tully-Fisher relation and where is the improvement of this relation over the normal relation?

PROBLEM 3

The Crab pulsar (PSR B0531+21) is a rapidly rotating neutron star in the center of the Crab Nebula, the remnant of a supernova observed in 1054 CE. The observed rotation period, mass, and radius of the pulsar is $P = 0.0333s$, $M = 1.4M_{\odot}$, and $R = 10$ km.

(a) (1 point) The "light cylinder" is the distance from the pulsar axis at which matter co-rotating with the magnetic field would (under classical assumptions) move at the speed of light. Derive an expression for the light cylinder radius, R_{LC} in terms of the rotation period P, and then calculate its numerical value for the Crab pulsar.

(b) (2 points) Calculate the minimum rotation period P_{min} (the "breakup period" corresponding to the maximum rotation rate) at which centrifugal forces at the equator would equal gravity, causing the neutron star to break apart.

(c) (2 points) Calculate the rotational kinetic energy of the Crab pulsar, E_{rot}. Express your answer in ergs and compare it to the total energy output of the Sun over one year. You can assume the moment of inertia is that for a sphere with constant density, $I = (2/5)MR^2$ (neutron stars are surprisingly close to uniform density).

(d) (4 points) The luminosity of the whole Crab Nebula is $L_{\mathrm{neb}} = 5 \times 10^{38}$ erg/s (across all wavelengths). Assuming the nebula is powered entirely by the loss of rotational energy from the pulsar, calculate the required rate of change of the pulsar's rotation period, $\dot{P} = dP/dt$, to supply the observed luminosity. Hint: Start with the rotational kinetic energy, and find dE_{rot}/dt in terms of $\dot{P}$.

(e) (1 point) If the pulsar were to continue spinning down at a *constant rate* of $\dot{P}$ you calculated in part (d), how long would it take for the rotation period to double from its current value? By comparing this with the age of the system, comment on the plausibility that the Crab nebula is powered entirely by the rotational kinetic energy of the pulsar.

PROBLEM 4

(a) (2 points) Describe the different components of the Milky Way galaxy and the spatial distribution of each component.

(b) (2 points) Compare the masses for each Galactic component described in (a).

(c) (1 point) What is a NFW profile?

(d) (3 points) Make a sketch of the chemical distribution of stars in the [α/Fe] vs [Fe/H] plane of the inner part of the Milky Way Galaxy ($R < 5$ kpc) and for stars in a 1 kpc sphere around the solar location. Identify key features and link them to the areas identified in (a).

(e) (2 points) Describe the key production sites for chemical elements, including stellar progenitors and timescales of formation.

PROBLEM 5

Suppose a main sequence star at a distance of d = 8 pc from the Earth has been observed to have a maximum radial velocity of 0.1 m/s, and the radial velocity varies periodically with a period P =1.5 years. From this we conclude that the star must have an unseen companion, a planet.

Assume that the star has $M = 0.8\ M_\odot$ and $T_{\rm eff} = 5000$ K. For simplicity, assume circular orbits and an inclination angle of 90°.

(a) (4 points) Calculate the average separation between the star and the planet.

(b) (4 points) Calculate the mass of the planet.

(c) (2 points) Can life exist on this planet? Explain.