ASTRONOMY QUALIFYING EXAM

January 2024

Notes and Instructions

• There are 5 problems. 4 of the 5 questions count as your grade on the exam. You may choose to answer only 4 questions, or the question with the lowest grade will be dropped. The other 4 questions will be used to grade the exam.
• Write on only one side of the paper for your solutions.
• Write your alias on every page of your solutions.
• Number each page of your solutions with the problem number and page number (e.g. Problem 3, p. 2/4 is the second of four pages for the solution to problem 3.)
• You must show your work to receive full credit.

Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol,\odot} = 4.74$ mag
$R_\odot = 7 \times 10^{10}$ cm
$T_{\rm eff,\odot} = 5777$ K
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
1 radian = 206265 arcsec
a = 7.56 $\times 10^{-15}$ erg cm^{-3} K^{-4}
c = 3 $\times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
k = 1.38 $\times 10^{-16}$ erg K^{-1} = 8.6173×10^{-5} eV K^{-1}
e = 4.8 $\times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
G = 6.67 $\times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s = 4.1357 $\times 10^{-15}$ eV s
1 amu = 1.66053886 $\times 10^{-24}$ g

PROBLEM 1

You observe a cluster of stars and notice from their positions in the H-R diagram that the 5 $M_\odot$ stars are just leaving the main sequence. You discover that one 5 $M_\odot$ star was formed of 80% hydrogen (by mass) and measure it to have a temperature of 14,000 K and luminosity of 300 $L_\odot$.

(a) (1 point) What is the radius of this 5 $M_\odot$ star, in units of solar radii?

(b) (3 points) Estimate the duration of the main sequence phase (in years) for this 5 $M_\odot$. Assume that it burns 10% of its hydrogen while on the main sequence.

(c) (3 points) Estimate the fraction of hydrogen with electrons in the $n = 2$ state, compared to that in the ground ($n = 1$) state, in the atmosphere of this star.

(d) (1 point) In the same cluster, you observe a star with 0.6 $M_\odot$. What is its age?

(e) (2 points) If the luminosity of the 0.6 $M_\odot$ star is 0.15 $L_\odot$, how much total hydrogen do you expect this star has today? Your estimate may be given as a fraction of the star's total mass, rounded to 2 significant figures. Explain your reasoning.

PROBLEM 2

The *Lucy* satellite recently discovered that the asteriod Dinkinesh has a moon. The moon is in a contact binary orbit, which means that the moon touches the surface of Dinkinesh. *Lucy* has a mass of 821 kg and it is moving with a velocity of 16,000 km/hour. The asteroid Dinkinesh has a diameter of 790 meters, and its moon has a diameter of 220 meters.

The masses of the asteriod and its moon are not known; you may assume that they have an average density of $2\,\mathrm{g\,cm^{-3}}$. For simplicity, you may assume that they are both spherical. Where appropriate below, you may assume that the masses of each body are concentrated at their centers.

(a) (2 points) Estimate the distance of the center of mass of the Dinkinesh/moon system from the surface of Dinkinesh.

(b) (3 points) What is the orbital period of the Dinknesh system?

(c) (3 points) NASA engineers rammed Lucy into the surface of Dinkinesh's moon. It struck exactly in the center of the moon, from behind, and ended up as debris on the surface of the moon. What is the resulting separation between Dinkinesh and its moon?

(d) (2 points) The amplitude of the light curve variability is 0.39 magnitudes. Keeping in mind that the emission of the system is simply reflected star light, and assuming a constant surface albedo, does this result agree with the assumption that Dinkinesh and its moon are spherical? Explain and show your work.

PROBLEM 3

(a) (1 point) Describe in a few sentences the Olbers' Paradox.

(b) (6 points) Assuming the universe is Euclidean with infinite volume and time, show that the total flux received by an observer from all stars in the universe will be infinite. Assume that all stars are the same with a radius R_*, temperature T_*, and number density of n_*.

(c) (3 points) List three possible solutions for the paradox with reasoning, based on the calculations performed in the previous part, on how the paradox is avoided.

PROBLEM 4

A beam of radiation with initial intensity I_0 enters an interstellar cloud with constant density, opacity, and emissivity.

(a) (4 points) Calculate $I_\nu(z)$.

(b) (3 points) What is its value at $\tau = 1$ and 2, if the source function is $S_\nu = 0.1 I_0$.

(c) (3 points) What is its value at large ($\tau >> 1$) optical depth.

PROBLEM 5

Assume that most of the mass of the Milky Way interior to the solar circle (with radius R) is in a spherical dark matter halo.

(a) (3 points) If the rotation curve is flat with V = 220 km/s, what is the local density of dark matter?

(b) (3 points) Use your result to estimate the mass of the dark matter within the solar system (out to Neptune's orbit of 30 AU). Do you expect the dark matter have a noticeable impact on planetary dynamics?

(c) (3 points) The baryonic mass of the Milky Way is about $6 \times 10^{10} M_\odot$. Assuming this is interior to the solar circle, what should the rotation velocity at the solar radius be in the absence of dark matter?

(d) (1 point) What is the NFW profile? Compare the sizes of the Milky Way's dark matter halo and its stellar halo.