ASTRONOMY QUALIFYING EXAM
January 2023

Notes and Instructions

- There are 6 problems. Attempt them all as partial credit will be given.
- Write on only one side of the paper for your solutions.
- Write your alias on every page of your solutions.
- Number each page of your solutions with the problem number and page number (e.g. Problem 3, p. 2/4 is the second of four pages for the solution to problem 3.)
- You must show your work to receive full credit.

Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$ mag
$R_\odot = 7 \times 10^{10}$ cm
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
1 radian = 206265 arcsec
a = 7.56 $\times 10^{-15}$ erg cm^{-3} K^{-4}
c = 3 $\times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
k = 1.38 $\times 10^{-16}$ erg K^{-1} = 8.6173×10^{-5} eV K^{-1}
e = 4.8 $\times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
G = 6.67 $\times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s = 4.1357 $\times 10^{-15}$ eV s
1 amu = 1.66053886 $\times 10^{-24}$ g

PROBLEM 1

Radiation of flux $I_{\nu_1}(0)$ is emitted by a nebula and travels toward an observer located at distance d from the nebula. Most of the space between the source and the observer is a vacuum. However, along the way the radiation travels a distance x through a cloud containing N dust grains per cm^{-3}, each grain with an absorption cross-section a_{ν_1}. When the light reaches the observer it has flux $I_{\nu_1}(d)$. **Assume that the source function over all of d, at all frequencies, is zero.**

(a) (3 points) Write down the appropriate equation of transfer (a differential equation), and solve it so that $I_{\nu_1}(d)$ is expressed in terms of the above quantities. Make sure to say what units you assume for a_{ν_1} (there is 1 choice that is easier than the other possibilites) and check that your units agree.

(b) (2 points) Show that the numerical increase in magnitude Δm due to extinction behaves linearly with respect to a_{ν_1}.

(c) (3 points) Consider transmission through the nebula at frequency ν_2, where $\nu_2 < \nu_1$. Note that the absorption cross-sections at the two frequencies will be different, and that generally $a \propto \nu$. Find an expression which relates the ratio of the fluxes I_{ν_2}/I_{ν_1} to the properties of the cloud containing the dust and the absorption cross-sections.

(d) (2 points) Sketch a graph of the logarithm (you can choose the most convenient base for log) of the flux ratio versus ν. Explain how/if your result is related to interstellar reddening.

PROBLEM 2

Consider a supernova observed in our Galaxy with an apparent magnitude of $m_V = -5$ and an absolute magnitude of $M_V = -20$ at the same point on the light curve. At peak brightness the supernova has $m_V = -6$ mag.

(a) (2 points) What is the ratio of the luminosity of the supernova *at maximum* to the solar luminosity? ($M_{V\odot} = +4.83$ mag).

(b) (2 points) Calculate the distance to the supernova (in parsecs), assuming that the color excess, $E(B-V) = 0.8$. Assume $R_V = 3.1$.

(c) (2 points) Now suppose the supernova was actually at a distance of 10 pc from the sun with the same absolute magnitude. How much brighter would the supernova appear than the full Moon, which has an apparent magnitude of -12.5? (Ignore any interstellar absorption here.) What would be the consequences if the supernova was actually at a distance of 10 pc?

(d) (2 points) Assuming that the supernova ejects 2 $M_\odot$ at a velocity of 10^4 km s^{-1}, how much kinetic energy (in ergs) is released? Using the distance you found in b), how long would it take (in years) for the ejected matter to reach us, assuming a constant velocity?

(e) (2 points) How many solar masses would have to be converted from H to He in nuclear fusion to produce the same kinetic energy you found above? (Assume 0.7% conversion.) Now compare the kinetic energy you found with the binding energy of a 2 $M_\odot$ neutron star with a radius of 10 km. How might we detect the release of such an energy from a supernova explosion?

PROBLEM 3

(a) (3 points) Describe the Pre Main Sequence (PMS) contraction of a 1 $M_\odot$ gas cloud up to the ZAMS stage, including a discussion of the temperature, dynamical, and radiative-transfer properties of the collapse. Draw the path in the HR diagram. What is the name of this part of the HR diagram? Why is the path here and not in some other part of the HR diagram? Relate what is happening inside of the PMS object to its observable parameters in the HR diagram.

(b) (3 points) Describe the evolution of a 5 $M_\odot$ star from the time it arrives on the main sequence until it reaches the top of the second giant branch (or AGB). In particular, give the position in the HR diagram at various stages. Describe in detail the physics of the red giant phase (first ascent of the giant branch). What is the final fate of this star? How do we know?

(c) (2 points) How does the evolution of a 5 $M_\odot$ star differ from that of a 1 $M_\odot$ and 25 $M_\odot$ star? Compare the evolution of a 1 $M_\odot$ star with solar metallicity to that of a 1 $M_\odot$ star with low metallicity (*i.e.* a Pop II star). What are the final fates of stars of 1 and 25 $M_\odot$?

(d) (2 points) Assuming that $(L/L_\odot) = (M/M_\odot)^\alpha$ where $\alpha = 3$, estimate the time spent on the main sequence for the 1, 5 and 25 $M_\odot$ stars. Describe the structure (*e.g.* the location of the convection and radiation zones) of these three stars while on the main sequence. Include a discussion of the energy sources in these stars.

PROBLEM 4

Briefly explain the following astronomical terms.

(a) (1 point) Age-metallicity relationship

(b) (1 point) K-correction

(c) (1 point) Luminosity function

(d) (1 point) Lyman alpha forest

(e) (1 point) Butcher-Oemler effect

(f) (1 point) Sunyaev-Zeldovich effect

(g) (1 point) de Vaucouleurs profile

(h) (1 point) Lin-Shu Density Wave Theory

(i) (1 point) Faber-Jackson Relation

(j) (1 point) Fanaroff-Riley luminosity classes

PROBLEM 5

Use $R_0 = 8$ kpc, $V_0 = 220$ km/s, and $c = 299792$ km/s in this problem, where R_0 is the Sun's distance to the Galactic center, and V_0 is its rotational speed around the Galactic center.

(a) (1 point) What transition is responsible for the 21cm emission from neutral Hydrogen atoms?

(b) (1 point) The more precise wavelength of the 21cm emission is 21.1061cm. Calculate the frequency (in MHz) of this emission.

(c) (3 points) We measure Doppler shifts of the 21 cm emission line from a series of gas clouds located in the Galactic disk along a line at a Galactic Longitude of $l = 30°$. We find that the minimum frequency of the 21cm emission from these clouds is at 1419.53 MHz. Draw a schematic diagram, and explain what causes this minimum frequency emission.

(d) (3 points) Calculate the rotational velocity of this cloud (in km/s) and its distance to the Galactic center (in kpc).

(e) (2 points) What is the mass of the Galaxy enclosed within the cloud (in Solar masses)?

PROBLEM 6

(a) (4 points) Consider a spherical dust grain in orbit about the Sun that is subjected to the Sun's gravity and radiation pressure. Assuming the object is perfectly absorptive, derive β, the force felt due to radiation pressure divided by that felt due to the Sun's gravity. Your answer should be in terms of the Sun's mass (M) and luminosity (L) as well as the grain's density (ρ) and radius (r).

(b) (2 points) Your answer in part a) should show that β depends on r^{-1}. In reality, β does not go to infinity as r gets smaller and smaller. In fact, β drops sharply for grain radii below ~1000 Angstroms. Why?

(c) (4 points) What is the maximum value of β that a dust grain lost by a comet on a circular orbit can have and still remain bound to the Sun? Since the comet's escape velocity is small, assume the grain's orbital velocity still matches that of the comet when it is initially lost from the comet.