Astro Qualifier Jan. 2021

- There are 6 problems. Attempt them all as partial credits will be given.
- Write on only one side of the provided paper for your solutions.
- Write your alias (NOT YOUR REAL NAME) on the top of every page of your solutions.
- Number each page of your solution with the problem number and page number (e.g. Problem 3, p. 2 is the second page for the solution to problem 3.)
- Do not staple your exam when done.
- You must show your work to receive full credit.

Constants:

$G = 6.67259 \times 10^{-8}$ dyne cm^2 g^{-2}
$c = 2.99792458 \times 10^{10}$ cm s^{-1}
$h = 6.6260755 \times 10^{-27}$ erg s
$k = 1.380658 \times 10^{-16}$ erg K^{-1}
$\sigma = 5.67051 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}
$m_p = 1.6726231 \times 10^{-24}$ g
$m_n = 1.674929 \times 10^{-24}$ g
$m_e = 9.1093897 \times 10^{-28}$ g
$m_H = 1.673534 \times 10^{-24}$ g
$e = 4.803206 \times 10^{-10}$ esu
1 eV $= 1.60217733 \times 10^{-12}$ erg
1 $M_\odot = 1.989 \times 10^{33}$ g
1 $L_\odot = 3.826 \times 10^{38}$ erg s^{-1}
1 pc $= 3.0857 \times 10^{18}$ cm
1 AU $= 1.4960 \times 10^{13}$ cm

1. Define/explain 10 of the following and indicate their relevance in astronomy. We expect that the answers should be less than 3–4 sentences to demonstrate your understanding of each item. Be sure to clearly indicate which of the 10 you would like graded (otherwise the 1st 10 items will automatically be graded).

   a) Initial Mass Function (1 point)
   b) Optical depth (1 point)
   c) Jean's radius (1 point)
   d) Flat field calibrations (1 point)
   e) Proper Motion (1 point)
   f) SN Ia (1 point)
   g) Cosmological principle (1 point)
   h) Airmass (1 point)
   i) SN II (1 point)
   j) Nova (1 point)
   k) Brown dwarf (1 point)
   l) Microlensing (1 point)

2. For this problem, assume that the Milky Way has radius $R_{MW} = 10^4$ pc, a scale height of h= 10^3 pc and has a Salpeter IMF.

   (a) (4 points) How many stars does one expect to find within 100 pc of the sun?

   (b) (6 points) If all stars are distributed evenly across the galaxy, how many of these will be B spectral type or earlier? Recall that B stars range in mass from 8-100$M_\odot$ and have an average lifetime (τ_B) of 10^7 years.

3. The Milky Way galaxy's black hole has been in the news lately. Here are some facts about that black hole.

- The mass has been determined to be about $4 \times 10^6\, M_\odot$.
- The galactic center lies about 8 kpc from the earth.
- The extinction in the V band is about 25 magnitudes.

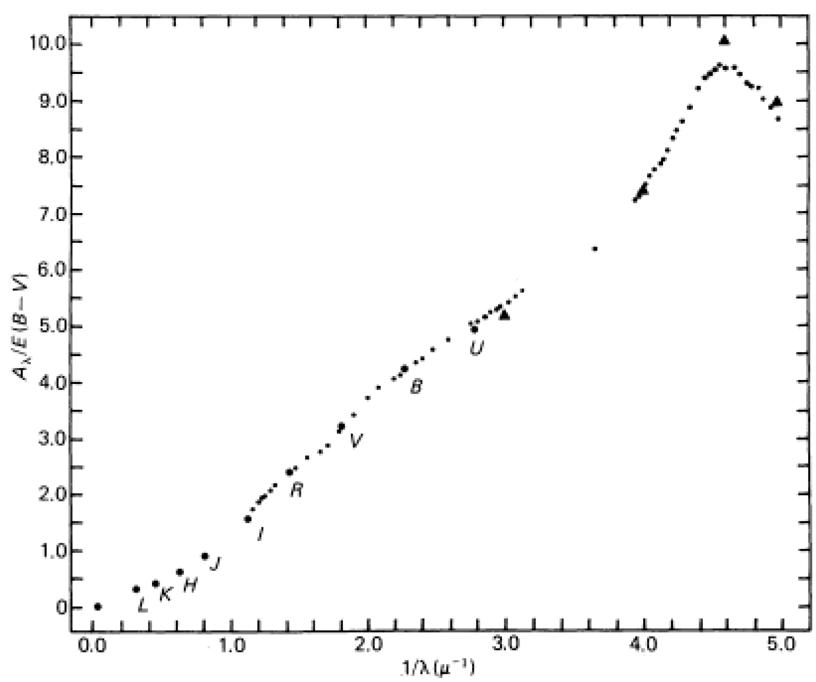
(a) (2 points) A new star was discovered among the stars located close to the black hole. It has orbital properties including a semi-major axis equal to 2 arc seconds and eccentricity of 0.75. You may assume that the orbit is in the plane of the sky. What is the distance to the black hole at closest approach in parsecs?

(b) (2 points) What is the period in years?

(c) (2 points) What is the velocity at closest approach in kilometers per second?

(d) (2 points) The star has an observed K-band magnitude of 14.0. What would its observed V-band magnitude be? Assume the star is a B0V star, with intrinsic V-K color of −0.40.

(e) (2 points) What is the intrinsic K-band magnitude of the star, with the extinction curve provided below?



4. (a) (1 point) What is the definition of r_{200} of a galaxy cluster?
   (b) (1 point) Explain the velocity dispersion of a galaxy cluster?
   (c) (1 point) What is the relation between the intrinsic velocity dispersion and the observed one?
   (d) (4 points) The mass inside r_{200} is considered as virialized. Using the virial theorem, derive a relation between the mass of the cluster M_{vir}, intrinsic velocity dispersion σ, and r_{200}. Assuming that the total mass including dark matter is all contained in member galaxies, $M_{vir} = \sum_i m_i$, where m_i is the mass of each galaxy, and r_{200} is close to the time averaged value of $2M_{vir}^2(\sum_{i \neq j} m_i m_j / r_{ij})^{-1}$.
   (e) (3 points) Further show the following theoretical prediction (given $\rho_{crit} = 3H_0^2/(8\pi G)$),

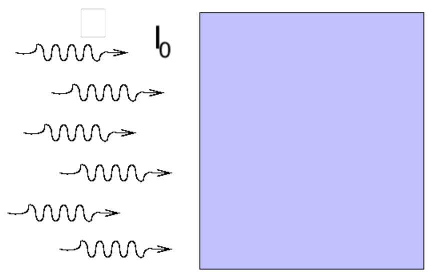
$$M_{vir} = \frac{\sigma^3}{10 H_0 G}. \tag{1}$$

5. For a star with mass M, radius R, and central density ρ_c, assume that the density changes as a function of radial distance r from the center as

$$\rho = \rho_c \left(1 - \left(\frac{r}{R}\right)^2\right) \tag{2}$$

   (a) (2 points) Find $m(r)$.
   (b) (1 point) Derive a mass-radius relation.
   (c) (3 points) Find an expression for the central pressure P_c (in terms of M, R, and constants only).
   (d) (2 points) Assuming that this star has an ideal gas equation of state, find the central temperature T_c.
   (e) (2 points) For a star with $M = M_\odot$ and $R = R_\odot$ (also assuming that the interior is mostly ionized hydrogen), what is the central temperature?

6. Consider a uniform slab of thickness T, in the z direction (horizontal direction in the figure). The left edge of the slab is sitting at the origin of the z-axis and the slab extends to infinity in the x and y directions. For this problem we will ignore emission from the slab itself. The slab is illuminated with a specific intensity I_0 at $z = 0$.



(a) (2 points) Consider the case of only pure absorption (specified by an opacity κ). Write down the equation of radiative transfer along the z-axis and solve for the emergent intensity at $z = T$

(b) (2 points) Now assume that we have both absorption and scattering (specified by a scattering opacity σ). Write down the equation for specific intensity I along the z-axis in this case.

(c) (3 points) Describe in words why the equation in **part:b** is much harder to solve than the one in **part:a**.

(d) (3 points) Describe the classical Lambda-iteration method for solving the scattering problem and the way in which it fails.