ASTRONOMY QUALIFYING EXAM
January 2012

Possibly Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
a = 7.56 $\times 10^{-15}$ erg cm^{-3} K^{-4}
c = 3 $\times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
k = 1.38 $\times 10^{-16}$ erg K^{-1}
e = 4.8 $\times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
G = 6.67 $\times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s
1 amu = 1.66053886 $\times 10^{-24}$ g

PROBLEM 1

A star of magnitude 0 delivers a flux density equal to $4.17 \times 10^{-11}\,\mathrm{erg\,s^{-1}\,cm^{-2}\,\AA^{-1}}$ in the K band ($\lambda = 2.2\mu$m).

a. Derive the flux density in units of $\mathrm{W\,m^{-2}\,Hz^{-1}}$ (2 points).

b. What is the count rate in terms of $\mathrm{photons\,s^{-1}\,cm^{-2}\AA^{-1}}$? (2 points)

c. What will be the diameter of a telescope whose diffraction limit at this wavelength is 0.05 arcsec? (2 points)

d. The telescope in part c has a focal ratio of 2. What would the size of a pixel in the detector have to be to critically sample the diffraction limit (NB: critically sampled means that the airy disk FWHM subtends two pixels)? (2 points)

e. The sky background at this wavelength is about $13.7\,\mathrm{mag\,arcsec^{-2}}$. Assuming that the detector and telescope present a quantum efficiency of 50%, what is the background rate per pixel for the detector imagined in part d? Assume that you are observing through a filter that has a width of $0.3\,\mu$m. (2 points)

PROBLEM 2

When a $5M_\odot$ star leaves the main sequence it enters the largely horizontal sub-giant branch. Models indicate that the star spends about 350,000 years on this section of the HR diagram before beginning its ascent on the red giant branch. Compute the expected Kelvin-Helmholtz time scale for this phase of stellar evolution and explain any differences by doing the following:

a. (4) Show that the gravitational energy ultimately radiated away is:

$$E_g = \frac{3GM^2}{10R},$$

where M and R are the the stellar mass and radius, respectively. Assume the virial theorem and that the density of the star at any distance from its center is equal to the star's average density, $M/\frac{4}{3}\pi R^3$.

b. (3) If $L = 10^3 L_\odot$ and $T_{eff} = 10^{3.9}$ K, estimate the time in years that this luminosity could be sustained if it is based solely on gravitational energy.

c. (3) Compare your answer in b. with the model-predicted time and explain why they are different. Make sure you explain what current theory tells us is going on inside the star.

PROBLEM 3

This problem concerns the important 21-cm line in astrophysics and its production mechanism.

a. (1) Briefly discuss the physics of the 21 cm line. What causes it? What's happening at the atomic level? Mention what kind of interstellar environment (density, temperature, state of hydrogen) is associated with this line.

b. (2) Estimate a minimum temperature that is necessary to excite this line and compare with a typical temperature of the interstellar environment which you identified in a. Discuss.

c. (3) Assuming a 2-level configuration, i.e., a ground state and one excited state, write down a rate equation which takes collisional excitation, collisional deexcitation, and spontaneous deexcitation into account. Use $\mathrm{q_{up}}$ and $\mathrm{q_{down}}$ to represent excitation and deexcitation collisional rate coefficients, respectively, A to represent the spontaneous downward rate, and $\mathrm{N_g}$ and $\mathrm{N_{ex}}$ to represent volume densities of ground and excited states.The sum of all the rates should equal zero.

d. (2) Based on your equation in c., show that the volume emissivity of 21 cm radiation, ϵ_{21}, is given by $\epsilon_{\mathrm{21cm}} = (\mathrm{N_g})^2 \mathrm{q_{up}} \mathrm{E_{21cm}}$, where $\mathrm{E_{21cm}}$ is the energy associated with the transition. Assume that $\mathrm{q_{down}} << \mathrm{A}$.

e. (2) Suppose an interstellar cloud produces 21-cm radiation with an optical depth at its center of $\tau = 0.5$. The line's full width at half-maximum of the line $\Delta \mathrm{v} = 10$ km/s. Find the thickness of the cloud in parsecs if $\tau = 5.2 \times 10^{-23} \frac{\mathrm{N_{col}}}{\mathrm{T}\Delta\mathrm{v}}$, where $\mathrm{N_{col}}$ is column density in cm^2 and Δv is in km/s. Assume an order-of-magnitude temperature and density which is characteristic of the system that typically emits 21-cm radiation.

PROBLEM 4

(1) Define the Eddington luminosity. (2pts)

(2) Derive the Eddington luminosity by balancing the radiation force and gravity for an electron. (3pts)

(3) What is the Eddington luminosity for a $10^8 M_\odot$ AGN? (3pts)

(4) An AGN is observed to emit at a super-Eddington rate. What are the possible explanations? (2pts)

PROBLEM 5

The observed universe can be described by the cosmological parameters that include H_0, Ω_m, Ω_r, Ω_k, and Ω_X.

(1) Define what H_0, Ω_m, Ω_r, Ω_k, and Ω_X are. (1 pt)

(2) Write down the expansion rate of the universe as a function of redshift. (3 pts)

(3) Derive an expression for the age of the universe as a function of redshift z (2pts)

(4) Galaxies have been observed at $z \sim 8$. Estimate the age of the universe at $z = 8$. (4 pts)

PROBLEM 6

Consider a rigid satellite of mass m, radius r, and density ρ_m orbiting at a distance d from its massive primary planet of mass M, radius R, and density ρ_M (see the figure below).

a. (2 pts) Show that the angular speed of the satellite about the primary is $\omega = \sqrt{\frac{GM}{d^3}}$

b. (3 pts) Find the differences in the gravitational acceleration between the center of the satellite (point 1) and the outer edge (point 2) due to the primary. Also find the differences in the centripetal acceleration between these two points. Show that the combination of the two effects is

$$\approx \frac{3GMr}{d^3}$$

c. (3 pts) The satellite will be tidally disrupted if the acceleration found in (b) is larger than the satellite's self gravitational acceleration. Show that the disruption occurs at

$$d = r(\frac{3M}{m})^{(1/3)} = R(\frac{3\rho_M}{\rho_m})^{(1/3)}$$

d. (2 pts) Assuming that the Earth and the Moon have the same density, at what distance would the Moon be disrupted? What about a moon around an Earth-size $1M_\odot$ ($3 \times 10^6 M_{earth}$) white dwarf star?