ASTRONOMY QUALIFYING EXAM

January 2010

Possibly Useful Quantities

$L_\odot$ = 3.9 x 10^{33} erg s^{-1}
$M_\odot$ = 2 x 10^{33} g
$M_{bol\odot} = 4.74$
$R_\odot$ = 7 x 10^{10} cm
1 A.U. = 1.5 x 10^{13} cm
1 pc = 3.26 l.y. = 3.1 x 10^{18} cm
a = 7.56 x 10^{-15} erg cm^{-3} K^{-4}
c= 3.0 x 10^{10} cm s^{-1}
σ = ac/4 = 5.7 x 10^{-5} erg cm^{-2} K^{-4} s^{-1}
k = 1.38 x 10^{-16} erg K^{-1}
e = 4.8 x 10^{-10} esu
1 fermi = 10^{-13} cm
N_A = 6.02 x 10^{23} moles g^{-1}
G = 6.67 x 10^{-8} g^{-1} cm^3 s^{-2}
m_e=9.1 x 10^{-28} g
h = 6.63 x 10^{-27} erg s
1 amu = $1.66053886 \times 10^{-24}$ g

PROBLEM 1

Sirius A and B form a visual binary. The orbital period is 50 years and parallax indicates that their distance from Earth is 8.6 ly.

a. (1 pt) If the average angular separation is 7.56", what is the average projected separation of the stars, in AU?

b. (3 pts) Sirius A is an A1 star with an estimated mass of $2.02\,\mathrm{M}_\odot$. Assuming that the orbit is in the plane of the sky, what is the mass of Sirius B, in solar masses?

c. (3 pts) A fit to the spectrum of Sirius B indicates that it has a surface temperature of 24,800 K and a bolometric luminosity of $0.026\,\mathrm{L}_\odot$. What is the radius of Sirius B, in solar radii?

d. (3 pts) Although the stars are well separated on the sky, Sirius B is difficult to observe in the optical because it is comparatively faint. The surface temperature of Sirius A is 9880 K and the bolometric luminosity is $25.4\,L_\odot$. Assuming both stars have spectra that can be approximated as black bodies, determine the ratio of the flux of Sirius A to that of Sirius B at 5500Å.

PROBLEM 2

This question is about the evolution of the Sun.

a. (3 pts) For the main–sequence, first–red–giant, and asymptotic–giant phases, describe the nuclear energy sources, i.e., say (1) which element is fusing, (2) by which reaction chain or cycle, (3) what is the principle product, (4) whether the fusion is in the core or in a shell, and (5) the temperature (to order of magnitude) at which the fusion occurs.

b. (4 pts)

For each of the three evolutionary phases of part (a), provide rough estimates of the minimum mass (in solar masses) and the maximum radius (in solar radii) attained, and the time since zero–age main sequence at which the maximum radius is attained. If the maximum radius ever exceeds 1 A.U., will Earth necessarily find itself beneath the photosphere and be swallowed by the Sun?

c. (3 pts)

Describe the final state of the Sun. Provide estimates of the mass and radius (in solar units) and the mass density (in g cm^{-3}). Discuss the momentum distributions of the electrons and the ions, the mode of energy transport from interior to surface, and the reason for it.

PROBLEM 3

Consider Nova Scorpii 1994. Observations suggest a binary consisting of a subgiant star and a black hole.

a. (3 pts) Discuss how we know it is a binary and why the unseen companion is suspected to be a black hole. What are the assumptions?

b. (4 pts) An important discovery was the presence of overabundant alpha elements such as neon, oxygen, and silicon in the observed subgiant. These elements are in proportions far above solar and can not have been produced in the subgiant; they are produced only in explosive nucleosynthesis at temperatures of $\sim 10^9$ K. What does this suggest about the final evolution of the star that became a black hole?

c. (3 pts) Discuss a scenario for how this black hole formed. How might we test this scenario?

PROBLEM 4

Briefly define and discuss, in a few sentences, the relevance of the following terms to modern astronomy.

a. (1 pt) Wolf-Rayet stars

b. (1 pt) initial mass function

c. (1 pt) alpha elements

d. (1 pt) age-metallicity relation

e. (1 pt) RR Lyrae variable stars

f. (1 pt) s–process

g. (1 pt) G dwarf problem

h. (1 pt) Crab Nebula

i. (1 pt) thin disk

j. (1 pt) instantaneous recycling

PROBLEM 5

a. (2 pts) Write down the equation of radiative transfer for a plane-parallel atmosphere and define all the terms.

b. (3 pts) Assume that the atmosphere is semi-infinite and that there is no external irradiation at the surface. Introduce the optical depth τ and derive the formal solution for an upward–directed intensity beam $I^+(\tau)$ in terms of the source function $S(\tau)$.

c. (3 pts) Assume that $S(\tau) = a_0 + a_1\tau$ and find $I^+(\tau,\mu),\ J(\tau),$ and $\mathcal{F}(\tau)$ at $\tau = 0$.

d. (2 pts) Write $I^+(0,\mu)$ and $\mathcal{F}(0)$ in terms of $S(\tau)$. These are called the Eddington–Barbier relations.

PROBLEM 6

There is strong observational evidence that our universe is dominated by dark energy today. For this problem assume a fixed matter density fraction $\Omega_m \ \ (= \Omega_{baryon} + \Omega_{non-baryon})$ and a flat universe ($\Omega_{tot} = 1$).

a. (1 pt) According to the current "concordance cosmology", what are the values of Ω_{baryon}, $\Omega_{non-baryon}$, and $\Omega_{dark\ energy}$?

b. (3 pts) Assume that dark energy has a constant equation of state, w_X. What is its density as a function of redshift?

c. (1 pt) What is the most likely value of w_X according to current data, and what is its implication?

d. (5 pts) In general, $w_X < 0$. How does the comoving distance of an object at redshift z depend on the value of w_X?