Astronomy Qualifier - January 2007

Lots of necessary (and some unnecessary) "Constants" on last page!

Problem 1:

This question concerns two kinds of thermonuclear instabilities that occur in stars. The first one occurs when ignition of a nuclear fuel takes place in electron–degenerate matter.

a)[2 points] Explain in as much detail as you can, and with reference to the equation of state, why such an instability occurs.

b)[2 points] Is such an instability thought to have occurred in the past evolution of the sun? Is such an instability expected to occur in the future evolution of the sun? If you answer yes to either question, then discuss the circumstances in which it did or will occur, and discuss the consequences.

c)[2 points] Is such an instability thought to ever occur in white dwarfs? If yes, discuss the circumstances in which it occurs, and discuss the consequences.

The second kind of instability occurs when a nuclear fuel ignites in a geometrically thin shell of non–electron–degenerate matter.

d)[2 points] Explain in as much detail as you can, and with reference to the equation of state, why such an instability occurs.

e)[2 points] Is such an instability thought to have occurred in the past evolution of the sun? Is such an instability expected to occur in the future evolution of the sun? If you answer yes to either question, then discuss the circumstances in which it did or will occur, and discuss the consequences.

Problem 2:

Consider a uniform slab of thickness T, in the z direction. The slab is sitting at the origin of the z-axis and extends to inifinity in the x and y directions. The slab is illuminated with a specific intensity I_0 at $z = 0$.

a) [2 points] Ignoring emission from the slab itself, consider the case of only pure absorption (specified by an opacity κ). Write down the equation of radiative transfer along the z-axis and solve for the emergent intensity at $z = T$

b) [2 points] Now assume that the slab is in pure LTE with emission at temperature T. Write down the differential equation of transfer for I along the z-axis in this case.

c) [1 point] Generalize the equation of transfer in case (b) for all directions (not just parallel to the z-axis).

d) [5 points] Derive the condition of radiative equilibrium in case (c).

Problem 3:

a) [2 points] The free-fall acceleration is given by

$$\frac{|d^2R|}{|dt^2|} = g,$$

where R is radius, t time, and g the local gravity. Use dimensional analysis to get an expression for the free-fall time scale, τ_{f-f}, in terms of the density ρ and a constant. For a star of M = 2 $M_\odot$ and R = 2 $R_\odot$, calculate τ_{f-f}.

b) [2 points] Calculate the total gravitational potential energy available to this same star. Assume the luminosity of a 2 $M_\odot$ star is L = $(\frac{M}{M_\odot})^3 L_\odot$. For this star, calculate the Kelvin-Helmholtz (i.e., gravitational) time scale in years.

c) [4 points] Using dimensional analysis and the hydrostatic equilibrium equation (dP/dr = $-\rho$g, where P is pressure, r radius, ρ mass density and g the local gravity) write down an expression for the pressure in a star as a function of mass, radius and a constant. Set that expression equal to the equation of state for an ideal gas (dP/dr = ρkTN_A/μ, where k is the Boltzmann constant, N_A is Avogadro's number and μ is the mean molecular weight), in terms of density and temperature, etc., and derive an expression for the central temperature, T, of the star. Assume 100% (ionized) H gas (so $\mu = 1/2$) and determine numerically the central temperature of this same star. (Note: you do not need to know or calculate the average density; just assume that the average density is given by the standard approximation relating M and R.) Does this temperature make sense to you? Explain.

d) [2 points] Calculate the nuclear time scale for the same star assuming that only 10% of the star's mass contributes to energy generation and the luminosity is given in b). [Assume 0.7 % mass loss in the nuclear conversion.] Compare the three time scales you found for this star.

Problem 4:

Betelgeuse is a red supergiant star. Assume it has an absolute visual magnitude of -5.10 and has a V band bolometric correction of -1.80 mag.

a) [3 points] If Betelgeuse has an effective temperature of 3600 K, calculate the effective radius of Betelgeuse in terms of the radius of the Sun.

b) [3 points] Betelgeuse may soon (at least on an astronomical time scale!) explode as a Type II supernova. Assume that, at peak brightness, this supernova will be a factor of 43,000 brighter in V than the current V brightness of Beteleguse. Assume that Betelgeuse has a parallax of 8 milliarcseconds. What will be the peak apparent visual magnitude of the Beteleguse supernova as seen from Earth?

c) [1 point] Hydrogen is the dominant element in the outer layers of Betelgeuse. However, it would be difficult to see any spectral lines of hydrogen in the visible wavelength region of the spectrum. Carefully explain the atomic physics behind this statement.

c) [1 point] Wolf 360 is a dwarf star with the same spectral type as Betegeuse. What differences would there be in the appearance of the spectra of these two stars? What is the physics that explains these differences?

c) [2 points] Explain the essence of the physics of a Type II supernova.

Problem 5:

From the surface of the Earth, you want to shoot a projectile using a very large gun and have the projectile just reach as far as circular geosynchronous orbit (where an object orbits the Earth once a day). (Assume the projectile arrives at the geosyncronous orbit distance with precisely zero kinetic energy).

a) [3 points] Calculate the distance above the Earth's surface an object would be if it were in a circular geosynchronous orbit.

b) [3 points] What is the minimum energy per gram of projectile that would be required to get the projectile to this distance?

c) [4 points] At a height of 3.5E8 cm above the Earth's surface, a satellite will measure the vertical speed of the projectile to see if it is "on track" to get to the desired distance. Assuming the projectile was launched with the correct initial speed, what speed whould the satellite measure? (Ignore rotation of Earh and air resistance).

Problem 6:

The Robertson-Walker metric and the Einstein equations together give us the Friedmann equation:

$$(\frac{\dot{R}}{R})^2 + \frac{k}{R^2} = \frac{8\pi G\rho}{3}$$

as well as

$$(\frac{\ddot{R}}{R}) = -\frac{4\pi G}{3}(\rho + 3p)$$

a) [4 point]
i) Define the cosmological parameters H_0, Ω_m, and Ω_Λ.
ii) What kind of observational data are required for measuring these cosmological parameters?
iii) What are the values of H_0, Ω_m and Ω_Λ that are most consistent with current observational data?
iv) What do they tell us about the ultimate fate of the Universe?

b) [5 points] Derive the cosmic deceleration parameter today, q_0, in terms of cosmological parameters such as Ω_m and Ω_Λ.

c) [1 point] What does your answer to b) tell us about the expansion history of the universe?

CONSTANTS

$$\sigma = 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4};\ c = 3.00 \times 10^{10} \text{ cm s}^{-1}; T_\odot = 5,800\text{K}$$

$$G = 6.67 \times 10^{-8} \text{ g}^{-1} \text{ cm}^3 \text{ s}^{-2};\ k = 1.38 \times 10^{-16} \text{ erg K}^{-1}$$

$$m_H = 1.67 \times 10^{-24} \text{ g};\ m_e = 9.11 \times 10^{-28} \text{ g};\ M_\odot = 1.99 \times 10^{33} \text{ g}$$

$$M_{\text{earth}} = 5.97 \times 10^{27} \text{ g}; M_G = 4.0 \times 10^{11} M_\odot$$

$$h = 6.63 \times 10^{-27} \text{ erg s};\ a = 7.56 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}$$

$$R_\odot = 6.96 \times 10^{10} \text{ cm};\ R_{\text{earth}} = 6.37 \times 10^{8} \text{ cm}$$

$$1 \text{ AU} = 1.496 \times 10^{13} \text{ cm}$$

$$1 \text{ parsec} = 3.09 \times 10^{18} \text{ cm};\ 1\mathring{A} = 10^{-8} \text{ cm}$$

$$M_V(\odot) = 4.8;\ M_{bol}(\odot) = 4.7;\ L_\odot = 3.9 \times 10^{33} \text{ ergs s}^{-1}$$

$$1 \text{ year} = 3.16 \times 10^{7} \text{ s}$$