# ASTRONOMY QUALIFYING EXAM
# August 2023

*Notes and Instructions*

• There are 5 problems. 4 of the 5 questions count as your grade on the exam. You may choose to answer only 4 questions, or the question with the lowest grade will be dropped. The other 4 questions will be used to grade the exam.
• Write on only one side of the paper for your solutions.
• Write your alias on every page of your solutions.
• Number each page of your solutions with the problem number and page number (e.g. Problem 3, p. 2/4 is the second of four pages for the solution to problem 3.)
• You must show your work to receive full credit.

*Useful Quantities*

$L_\odot = 3.9 \times 10^{33}$ erg s$^{-1}$
$M_\odot = 2 \times 10^{33}$ g
$M_{bol,\odot} = 4.74$ mag
$R_\odot = 7 \times 10^{10}$ cm
$T_{\mathrm{eff},\odot} = 5777$ K
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
1 radian = 206265 arcsec
a = 7.56 $\times 10^{-15}$ erg cm$^{-3}$ K$^{-4}$
c = 3 $\times 10^{10}$ cm s$^{-1}$
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm$^{-2}$ K$^{-4}$ s$^{-1}$
k = 1.38 $\times 10^{-16}$ erg K$^{-1}$ = 8.6173 $\times 10^{-5}$ eV K$^{-1}$
e = 4.8 $\times 10^{-10}$ esu
1 fermi = $10^{-13}$ cm
$N_A = 6.02 \times 10^{23}$ moles g$^{-1}$
G = 6.67 $\times 10^{-8}$ g$^{-1}$ cm$^3$ s$^{-2}$
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s = 4.1357 $\times 10^{-15}$ eV s
1 amu = 1.66053886 $\times 10^{-24}$ g

## PROBLEM 1

Consider an A0 star. An example of an A0 star is Vega. Here are some facts about A0 stars:

- The surface temperature of an A0 star is 9800 K.
- The radius of an A0 star is 2.2 solar radii.
- The mass is 2.9 $M_\odot$.

(a) (1 point) Estimate the luminosity of an A0 star in terms of solar luminosities.

(b) (3 points) Using simple physical principles, estimate the central pressure of an A0 star.

(c) (3 points) Using simple physical principles, estimate the central temperature of an A0 star.

(d) (3 points) Estimate the hydrogen-burning lifetime of an A0 star. Assume that only the inner 10% is available for burning.

## PROBLEM 2

If the source function inside a star is $S_\nu(\tau_\nu) = a_\nu + b_\nu \tau_\nu$, where $a_\nu$ and $b_\nu$ are functions of $\nu$, calculate

(a) (4 points) the specific intensity $I_\nu(0, u)$ at the surface for the outgoing directions ($u \geq 0$)

(b) (2.5 points) the average intensity $J_\nu$

(c) (2.5 points) the Eddington Flux $H_\nu$

(d) (1 point) the monochromatic Flux $F_\nu$

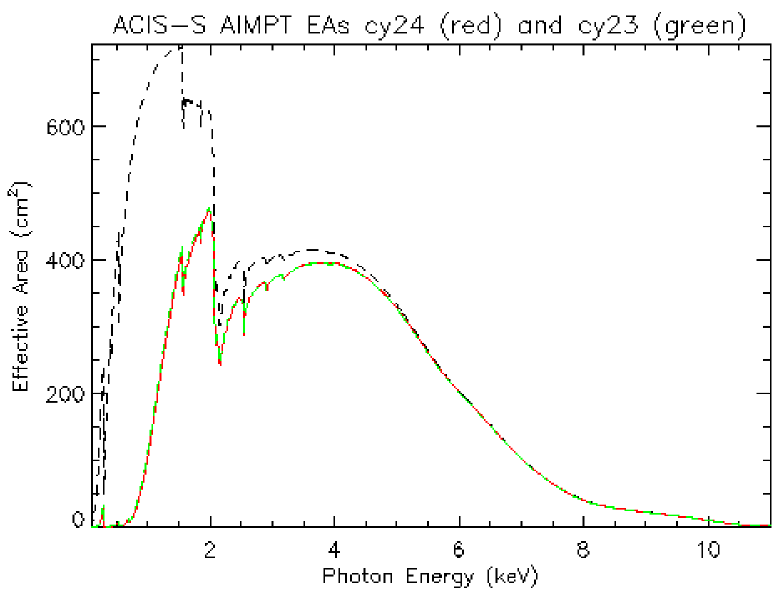
# PROBLEM 3

Consider the following general observational problem. A target with photon count rate $n$ photons $s^{-1}$ is observed for a time $T$. Extraction of the count rate in a source-free region shows there is a high and non-negligible background signal with a rate $b$ (also in photons $s^{-1}$). Therefore, the total signal in the target extraction region is $t = n + b$.

(a) (3 points) Derive an equation for the signal-to-noise ratio of the net target signal $n$ as a function of the exposure time, the background count rate $b$ and the total count rate $t$.

(b) (6 points) A theory that you have developed leads you to believe that the X-ray spectrum of a galactic black hole should have a Gaussian emission line located at 8 keV. The velocity width of the line is $10^4\,\mathrm{km\,s^{-1}}$, and the equivalent width of the line is 500 eV. There is also a continuum spectrum present that has a wavelength independent photon flux density of $1.0 \times 10^{-5}$ photons $\mathrm{s^{-2}\,cm^{-2}\,keV^{-1}}$ that spans the full X-ray bandpass.

How long would you need to observe the object using Chandra to detect the predicted emission with a signal-to-noise ratio of 3? A plot of the Chandra effective area curve is included below.




If you need to make assumptions to solve this problem, please clearly state what they are, label them as assumptions, and justify them.

(c) (1 point) Qualitatively, would the observation have to be longer or shorter to obtain the same signal-to-noise ratio for an emission line with a width of 1000 km/s? Explain.

## PROBLEM 4

(a) (1 point) Brifly explain the cosmic scale factor $a$.

(b) (1 point) What is the relation between the cosmic scale factor and redshift.

(c) (1 point) Briefly explain the term k-correction in extragalactic astronomy.

(d) (7 points) The specific flux and luminosity are related by the following equation

$$\mathsf{f}_\nu = \frac{\mathsf{L}_{\nu'}(1+\mathsf{z})}{4\pi \mathsf{D}_\mathsf{L}^2}, \qquad \nu' = \nu(1+\mathsf{z}). \tag{1}$$

Derive the monochromatic k-correction for a power-law spectrum $L_\nu \propto \nu^{-\alpha}$ in magnitude units.

## PROBLEM 5

In this problem, you will estimate the cooling timescale for dissipative collapse during the formation of protogalaxies. Assume that the collapsing protogalactic nebula has mass $M$, and radius $R$. To estimate this time scale, you must first determine the characteristic amount of thermal energy contained within each particle in the gas.

(a) (2 points) Using the viral theorem, assume the gas is in quasi-static equilibrium, and relate the thermal kinetic energy of the gas to the potential energy. Assume the gas has a mean molecular weight $\mu$, and contains $N$ particles (mass of hydrogen is $m_H$).

(b) (2 points) Recall that $\sigma$, the velocity dispersion, is related to gas velocity as $\sigma = \sqrt{< v^2 >}$. Solve for the velocity dispersion of the gas.

(c) (2 points) Determine a characteristic temperature of the gas, known as the viral temperature, by equating the typical kinetic energy of the gas to its thermal energy.

(d) (2 points) To estimate the cooling time, assume that the cooling rate for the gas (in units of ergs/s/cm$^3$) is $r_{cool} = n^2\Lambda(T)$, where n is the number density of particles in the gas and $\Lambda(T)$ is the quantum mechanical cooling function. Solve for $t_{cool}$ assuming all the energy of the cloud is radiated in away in that amount of time. Your answer should be in terms of the virial temperature, $\Lambda$ and $n$.

(e) (2 points) Suppose that the freefall time ($t_{ff}$) is less than the cooling time ($t_{cool}$). Discuss what this means in terms of the collapse of the protogalactic nebula.