Astro Qualifier Aug. 2021

- There are 6 problems. Attempt them all as partial credits will be given.
- Write on only one side of the provided paper for your solutions.
- Write your alias (NOT YOUR REAL NAME) on the top of every page of your solutions.
- Number each page of your solution with the problem number and page number (e.g. Problem 3, p. 2 is the second page for the solution to problem 3.)
- Do not staple your exam when done.
- You must show your work to receive full credit.

Constants:

$G = 6.67259 \times 10^{-8}$ dyne cm^2 g^{-2}
$c = 2.99792458 \times 10^{10}$ cm s^{-1}
$h = 6.6260755 \times 10^{-27}$ erg s
$k = 1.380658 \times 10^{-16}$ erg K^{-1}
$\sigma = 5.67051 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}
$m_p = 1.6726231 \times 10^{-24}$ g
$m_n = 1.674929 \times 10^{-24}$ g
$m_e = 9.1093897 \times 10^{-28}$ g
$m_H = 1.673534 \times 10^{-24}$ g
$e = 4.803206 \times 10^{-10}$ esu
1 eV $= 1.60217733 \times 10^{-12}$ erg
1 $M_\odot = 1.989 \times 10^{33}$ g
1 $L_\odot = 3.826 \times 10^{38}$ erg s^{-1}
1 $R_\odot = 6.96 \times 10^{10}$ cm
1 pc $= 3.0857 \times 10^{18}$ cm
1 AU $= 1.4960 \times 10^{13}$ cm

1. (a) (6 points) Describe within a few sentences the main physical picture of the processes involved for at least four sources of opacity important in stellar atmospheres. The description may include but not require diagrams.

   (b) (4 points) Explain why the solution of the Saha equation is necessary to calculate the opacities in Part (a). Restricting your discussion to hydrogen, what quantities are needed to calculate each opacity source (not including physical constants)?

2. A beam of radiation with an initial intensity $I_{\nu,0}$ enters an interstellar cloud with constant density, opacity, and emissivity, where ν is the frequency of the light.

   (a) (4 points) Calculate the intensity $I_\nu(z)$ as the beam of radiation travels in the z direction in the interstellar cloud.

   (b) (3 points) What is the value of $I_\nu(z)$ at the optical depths of $\tau = 1$ and 2, if the source function is $S_\nu = 0.5I_0$.

   (c) (3 points) What is the value of $I_\nu(z)$ at large ($\tau >> 1$) optical depth.

3. (a) (3 points) Draw and label the different fundamental interior and atmospheric regions of the Sun.

   (b) (4 points) Use the Virial Theorem to estimate the approximate internal temperature of the Sun as a function of Sun mass and radius.

   (c) (2 points) Draw a graph that shows how the temperature profile of the Sun changes as a function of radius, from the core to the outer extent of the Solar atmosphere.

   (d) (1 point) Explain why the outer extent of the Solar atmosphere is so much hotter than the photosphere.

4. The surface brightness of an elliptical galaxy follows the de Vaucouleurs profile,

$$I(R) = I_e \exp\left\{-7.669\left[\left(\frac{R}{R_e}\right)^{1/4} - 1\right]\right\}, \tag{1}$$

where I is the surface brightness, R is the projected radius, R_e is the effective radius, I_e is the surface brightness at R_e. This profile has the property that the total luminosity is

$$L_{tot} = 7.215\pi I_e R_e^2. \tag{2}$$

The galaxy has a total B band luminosity of $M_B = -23$ mag. The effective radius of the galaxy is 4 kpc.

   (a) (2 points) Express the de Vaucouleurs profile in magnitude unit, $\mu(R)$, as a function of μ_e, R, and R_e, where μ_e is the surface brightness in magnitude unit at R_e.

(b) (1 point) What is the B band luminosity of the galaxy within the effective radius (in magnitude)?

(c) (2 points) The galaxy is located at 20 Mpc and the foreground extinction from the Milky Way is $E(B-V) = 0.122$ and the slope of Milky Way extinction curve is $R_V = 3.1$. Calculate the total apparent B band magnitude of this galaxy?

(d) (5 points) R_{25} is the radius, where the observed surface brightness is $\mu = 25$ mag arcsec^{-2}. Calculate the R_{25} radius for this galaxy (in kpc).

5. This question relates to finding the mass of a galaxy cluster, which you can assume is spherical and contains a large number of galaxies.

   (a) (6 points) Describe 3 different methods that can be used to determine the total mass of this galaxy cluster. (2 points/method)

   (b) (4 points) Derive an expression for the virial mass (M_{vir}) for a galaxy cluster, assuming a radius R_{vir}, a total of n galaxies of mass m within the cluster's virial radius, and assuming you have measured the velocity dispersion of the member galaxies σ_r. Your solution should only be in terms of R_{vir} and σ_r.

6. The Universe is dominated by dark energy today, but for a rough estimate of the age of the Universe at redshifts $2 < z < 100$, we can assume a matter dominated universe.

   The Friedman Equation for matter dominated universe is

$$\dot{R}^2 + k = \frac{8\pi G}{3}\rho R^2, \tag{3}$$

   where

$$\frac{8\pi G}{3}\rho = \Omega H^2, \tag{4}$$

   and R is the size of a spherical shell, k is the curvature of the universe, G is the gravitational constant, ρ is the matter density, Ω is the dimensionless density parameter for matter, and H is the Hubble parameter.

   (a) (5 points) Derive the formula for the age of a matter-dominated universe at redshift z, assuming that we know t_0 (the age of the Universe today).

   (b) (2 points) What is the current measurement of t_0? Estimate the age of the Universe at $z = 10$ using this information.

   (c) (3 points) How does dark energy change the age of the Universe today, compared to a flat universe with matter only?