ASTRONOMY QUALIFYING EXAM
August 2019

Possibly Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
1 radian = 206265 arcsec
a = 7.56 $\times 10^{-15}$ erg cm^{-3} K^{-4}
c = 3 $\times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
k = 1.38 $\times 10^{-16}$ erg K^{-1}
e = 4.8 $\times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
G = 6.67 $\times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s = 4.1357 $\times 10^{-15}$ eV s
1 amu = 1.66053886 $\times 10^{-24}$ g

PROBLEM 1

a) [3 pts] Calculate the bolometric solar flux at 1 AU from the Sun.

b) [3 pts] The average albedo of the Earth is about 0.3. What is the total amount of solar radiation absorbed by the Earth every second.

c) [3 pts] Assuming that the the Earth behaves as a blackbody, find its equilibrium temperature, $\mathrm{T_{eq}}$.

d) [1 pt] What environmental effect is responsible for raising Earth's average temperature above the value you have calculated?

PROBLEM 2

a) [3 pts] Imagine the TESS mission discovers a Sunlike star whose brightness periodically dims by 0.01%. If this is caused by a *circularly* orbiting planet transiting the star's *equator*, find the radius of this planet in Earth Radii.

b) [4 pts] If the transit duration is 6.5 hours, estimate the planet's semimajor axis. Drawing a picture of the transit may help.

c) [3 pts] Follow-up radial velocity measurements find the value of $\mathrm{m_p} \sin \mathrm{i}$ to be 0.25 $\mathrm{M_{Earth}}$. Estimate the planet's composition. In the context of our own solar system's formation, explain why this planet's composition is or isn't surprising.

PROBLEM 3

a) [5 pts] Calculate the baryon density (in g/cm^3) at the epoch of Big Bang Nucleosynthesis, which has a characteristic temperature of 10^9 K, given the critical density of the universe $\rho_{\mathrm{crit}} = 3\mathrm{H}_0^2/(8\pi \mathrm{G})$, and $\Omega_\mathrm{b}\mathrm{h}^2 \simeq 0.02$, $\mathrm{h} = 0.7$.

b) [5 pts] Calculate the energy density released from the Big Bang nucleosynthesis and compare with the energy density of the universe at the time.

PROBLEM 4

Consider the H^- ion, an important source of opacity in stars with low temperature photospheres. The binding energy for the extra electron is 0.754 eV.

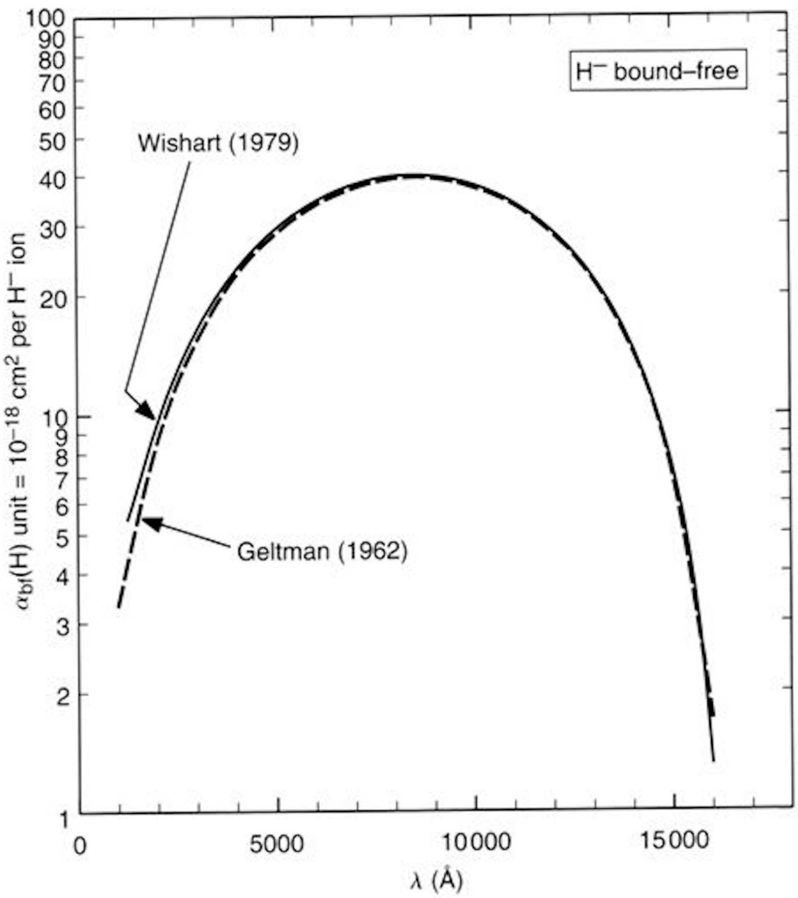
a) [2 pts] What wavelength photon can photoionize this ion?

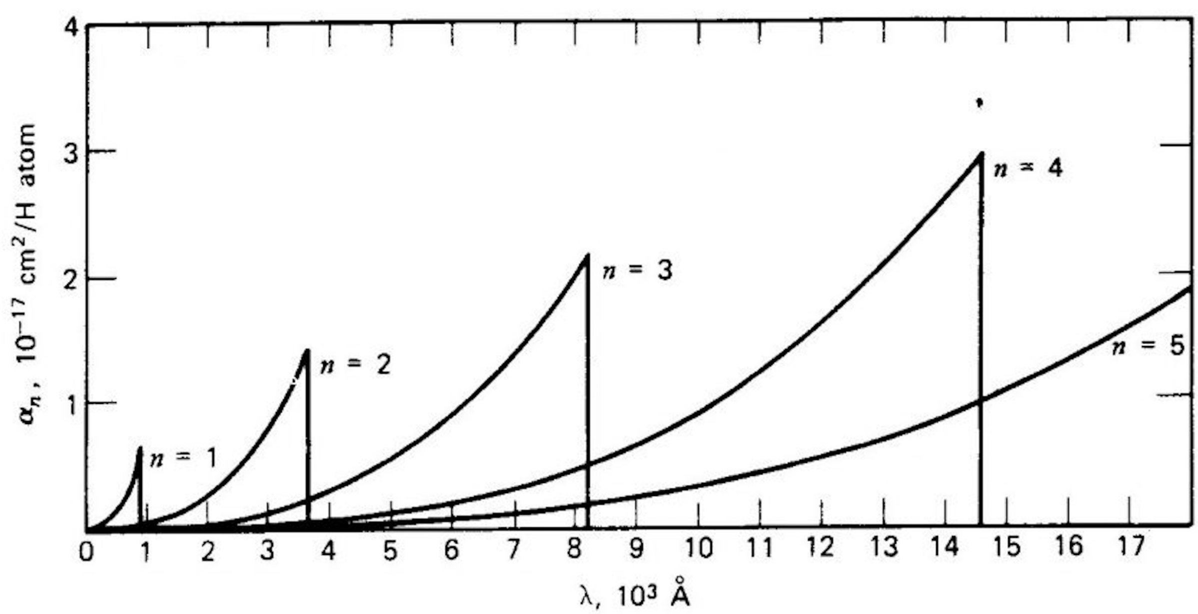
b) [2 pts] The opacity is important in atmospheres of stars with temperatures of about 4000 K. At this temperature, are there a lot of photons available that can ionize H^-? Explain and show your work.

c) [2 pts] Another complication is that at low temperatures, most atoms are neutral and there aren?t many electrons available to join onto hydrogen to form the H^-. Such electrons come from easily-ionzed atoms such as sodium. Sodium has an ionization potential of 5.139 eV. At a temperature of 4000 K, are there a lot of photons available that can ionize sodium? Show your work and discuss.

d) [2 pts] Collisional ionization of this ion may also be an issue. At 4000 K, do you expect a lot of electrons to be present that would be able to collisionally ionize the H^-? Show your work and explain your answer.

e) [2 pts] The plot of the bound-free cross section of this ion is attached. For comparison, the bound-free cross section of hydrogen is also attached. At 8000Å, the H^- cross section is high, but so is the n = 3 cross section of hydrogen. Which do you expect to dominate in the atmosphere of a star with a temperature of 4000K? How about a star with a temperature of 10000 K? Explain.


H⁻ bound–free
Wishart (1979)
Geltman (1962)
$\alpha_{bf}(H)$ unit = 10^{-18} cm^2 per H$^-$ ion
λ (Å)
0
5000
10000
15000


n = 1
n = 2
n = 3
n = 4
n = 5
α_n, 10^{-17} cm^2/H atom
λ, 10^3 Å

PROBLEM 5

a) [3 pts.] Describe the Pre Main Sequence (PMS) contraction of a 1 $M_\odot$ gas cloud up to the ZAMS stage, including a discussion of the temperature, dynamical, and radiative-transfer properties of the collapse. Draw the path in the HR diagram. What is the name of this part of the HR diagram? Why is the path here and not in some other part of the HR diagram? Relate what is happening inside of the PMS object to its observable parameters in the HR diagram.

b) [3 pts.] Now describe the evolution of a 5 $M_\odot$ star from the time it arrives on the main sequence until it reaches the top of the second giant branch (or AGB). In particular, give the position in the HR diagram at various stages. Describe in detail the physics of the red giant phase (first ascent of the giant branch). What is the final fate of this star? How do we know?

c) [2 pts.] How does the evolution of a 5 $M_\odot$ star differ from that of a 1 $M_\odot$ and 25 $M_\odot$ star? Compare the evolution of a 1 $M_\odot$ star with solar metallicity to that of a 1 $M_\odot$ star with low metallicity (*i.e.* a Pop II star). What are the final fates of stars of 1 and 25 $M_\odot$?

d) [2 pts.] Assuming that $(L/L_\odot) = (M/M_\odot)^\alpha$ where $\alpha = 3$, estimate the time spent on the main sequence for the 1, 5 and 25 $M_\odot$ stars. Describe the structure (*e.g.* the location of the convection and radiation zones) of these three stars while on the main sequence. Do not forget to indicate the energy sources in these stars.

PROBLEM 6

Assume that most of the mass of the Milky Way interior to the solar circle (with radius R) is in a spherical dark matter halo.

a) [3 pts.] If the rotation curve is flat with V = 220 km/s, what is the local density of dark matter?

b) [3 pts.] Use your result to estimate the mass of the dark matter within the solar system (out to Neptune's orbit of 30 AU). Do you expect the dark matter have a noticeable impact on planetary dynamics?

c) [3 pts.] The baryonic mass of the Milky Way is about $6 \times 10^{10} M_{\odot}$. Assuming this is interior to the solar circle, what should the rotation velocity at the solar radius be in the absence of dark matter?

d) [1 pt.] What is the NFW profile? Compare the sizes of the Milky Way's dark matter halo and its stellar halo.