Astro Qualifier August 2016

- There are 6 problems. Attempt them all as partial credits will be given.
- Write on only one side of the provided paper for your solutions.
- Write your alias (NOT YOUR REAL NAME) on the top of every page of your solutions.
- Number each page of your solution with the problem number and page number (e.g. Problem 3, p. 2 is the second page for the solution to problem 3.)
- Do not staple your exam when done.
- You must show your work to receive full credit.

Constants:

$G = 6.67259 \times 10^{-8}$ dyne cm^2 g^{-2}
$c = 2.99792458 \times 10^{10}$ cm s^{-1}
$h = 6.6260755 \times 10^{-27}$ erg s
$k = 1.380658 \times 10^{-16}$ erg K^{-1}
$\sigma = 5.67051 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}
$m_p = 1.6726231 \times 10^{-24}$ g
$m_n = 1.674929 \times 10^{-24}$ g
$m_e = 9.1093897 \times 10^{-28}$ g
$m_H = 1.673534 \times 10^{-24}$ g
$e = 4.803206 \times 10^{-10}$ esu
1 eV $= 1.60217733 \times 10^{-12}$ erg
1 $M_\odot = 1.989 \times 10^{33}$ g
1 $L_\odot = 3.826 \times 10^{38}$ erg s^{-1}
1 pc $= 3.0857 \times 10^{18}$ cm
1 AU $= 1.4960 \times 10^{13}$ cm
$H_0 = 70$ km s^{-1} Mpc^{-1}

1. The following problem refer to objects of relatively low density such as H II regions. Assume a two level atom with one bound electron. The atom is not hydrogen or helium.

   (a) (3 points) If N_1, N_2, and N_e are number densities of atomic levels 1, 2, and free electrons, respectively, and q_{12}, q_{21}, and A_{21} are the collisional excitation, collisional de-excitation, and spontaneous rate coefficients between levels 1 and 2, derive an expression for N_2/N_1 in terms of the other quantities.

   (b) (1 point) Explain what is meant by *critical density* and write down an expression for it.

   (c) (3 points) Write down an equation involving the excitation energies of the two levels for N_2/N_1 in the limiting case when electron density tends toward infinity. Explain.

   (d) (3 points) Write down an expression involving the quantities in a. for the energy emitted in a forbidden line per second per cm^3. By considering this expression as N_e approaches zero and approaches infinity (the two extreme cases) explain why the increase in the energy production rate of this radiation slows as the density rises.

2. A beam of radiation with initial intensity I_0 enters an interstellar cloud with constant density (ρ), opacity (κ_ν), and emissivity (j_ν).

   (a) (4 points) Using the radiative transfer equation (and assuming $u = cos\theta = 1$)

$$\frac{u}{\rho}\frac{dI_\nu(z,u)}{dz} = -\kappa_\nu I_\nu(z,u) + j_\nu. \tag{1}$$

   Calculate $I_\nu(z)$.

   (b) (3 points) What is its value at $\tau = 1$ and 2, if the source function is $S_\nu = 2I_0$.

   (c) (3 points) What is its value at large ($\tau \gg 1$) optical depth. Physically interpret this result.

3. The expansion equation is expressed as:

$$H^2(t) = H_0^2(\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda). \tag{2}$$

   (a) (1 point) Define a and H.

   (b) (1 point) What is the relation between a and the cosmic redshift z.

   (c) (4.5 points) Assume that we are in an Einstein de Sitter universe (a flat universe with only matter), DERIVE the formula for the luminosity distance.

   (d) (0.5 points) Calculate the luminosity distance to an object at $z = 5$.

   (e) (2 points) Calculate the angular diameter distance of that object.

   (f) (1 point) We observe the angular size of the object to be 1 arcsec. What is the physical size of the object (in kpc).

4. The APO 3.5m telescope instrument suite contains two visible wavelength spectrographs that both use diffraction gratings to achieve wavelength dispersion. The ARC Echelle Spectrograph (ARCES) is a R$\sim$31,500 instrument that provides simultaneous wavelength coverage between 4000-9000 Angstroms while the Dual Imaging Spectrograph (DIS) is a long-slit spectrograph that provides similar wavelength coverage with R$\sim$1500.

   (a) (3 points) Sketch and briefly describe the basic configuration of both ARCES and DIS (or analogous instruments). Also, sketch and briefly describe how a stellar spectrum would appear on the CCD(s) of each instrument.

   (b) (3 points) For the Hα line, calculate the $\Delta\lambda$ values associated with each of the following processes for an A-type star (i.e. Vega) having a rotational velocity of 100 km/s. State all assumptions that you make about necessary physical quantities such as temperature, etc:

   i) thermal broadening ii) rotational broadening

   (c) (1 point) Would either ARCES or DIS be appropriate tools to explore each of these physical processes? Justify briefly why or why not for each process

   (d) (3 points) Describe the type of calibration data you would need to take to analyze data from ARCES or DIS, and the physical purpose of each type of calibration.

5. An M4 star has a mass of 0.2 $M_\odot$, a radius of 0.26 $R_\odot$, and an effective temperature of 3100 K. Assume there is a 1 $M_\oplus$, 1 $R_\oplus$ planet in orbit around it, and that the star is the only energy source for the planet. Furthermore, assume that all the stellar radiation that strikes the planet is completely absorbed and redistributed evenly around the planet. Finally, assume that the star and planet are perfect blackbodies.

   (a) (6 points) Find the temperature of the planet if it is in a circular orbit at 1 AU.

   (b) (2 points) Define the inner edge of the habitable zone as the point at which water boils on the surface of a planet and the outer edge of the habitable zone as the point at which water freezes on the surface of a planet. Find the location (in AU) of the inner and outer edges of the habitable zone for the star-planet system from part a.

   (c) (1 point) Explain how a non-zero planetary albedo would affect the location of the habitable zone.

   (d) (1 point) Explain how a carbon dioxide atmosphere would affect the location of the habitable zone.

6. (a) (2 points) Define A_V, $E(B-V)$, and R_V of the interstellar medium.

   (b) (3 points) An unusual reddening curve has been derived from an HST observation of the active galaxy WPVS 007 (solid line below; Leighly et al. 2009), as compared to the SMC curve (dashed line). Why is this reddening curve expressed in terms of $E(2000-V)$ rather than the usual $E(B-V)$? Remember that the B band is centered at 4400Å and the V band is centered at 5500Å.

   (c) (5 points) The ion He+ produces an emission line at 1640.4 Å, resulting from a transition from the 4th level to the third level, and another one at 4687 Å resulting from a transition from the third level to the second level. Atomic physics demands that the ratio of 1640.4 Å line to the 4687 Å line should be 6.6 under certain conditions. If the reddening in the AGN host galaxy is $E(2000 - V) = 0.1$, what ratio of these two lines do you expect to measure?