Astronomy Qualifier - August 2011

Lots of necessary (and some unnecessary) "constants" and possibly useful integrals at end.

Problem 1:

The inflationay theory of the very early Universe solves the horizon problem of standard cosmology.

a) [2 pts] What is the horizon problem?

b) [2 pts] Show that inflation solves the horizon problem if $a(t) \propto t^{\alpha}$ during inflation, with $\alpha > 1$.

c) [4 pts] Derive the requirement from inflation on the equation of state of the matter-energy in the universe.

d) [2 pts] Does any matter-energy component that has been studied in cosmology satisfy this requirement?

Problem 2:

a) [5 pts] Assume that a model for the dark matter halo of the Galaxy is:

$$\rho(r) = \frac{C_0}{(a^2 + r^2)},$$

where ρ is density, r is distance from the galactic center, and $a = 2.8$ kpc. Show that the amount of dark matter interior to a radius r is given by the expression:

$$M_r = 4\pi C_0 \left[r - a \; tan^{-1} \left(\frac{r}{a} \right) \right]$$

b) [2 pts] If 5.5×10^{11} $M_\odot$ of dark matter is located within 100 kpc of the Galactic center, determine C_0 in units of $M_\odot$/kpc. Repeat your calculation if 1.3×10^{12} $M_\odot$ is located within 230 kpc of the Galactic center.

c) [3 pts] Estimate the amount of dark matter (in solar masses) within a radius of 50 kpc of the Galactic center. Compare your answer to the mass of the stellar halo (choose a reasonable value for the latter).

Problem 3:

Consider an eclipsing spectroscopic binary with the following properties:

- Orbital period is 6.31 yr.
- Maximum radial velocities of Star A and Star B are 5.4 kms^{-1} and 22.4 km s^{-1}.
- Time period between first contact and minimum light is 0.58 d, and the length of the primary minimum is 0.64 d.
- The apparent bolometric magnitudes of the maximum, primary minimum, and secondary minimum are 5.40 magnitudes, 9.20 magnitudes, and 5.44 magnitudes, respectively.

Assuming circular orbits and that the plane of the system lies in our line of sight, find the following:

a) [2 pts] Ratio of the stellar masses.

b) [2 pts] Sum of the masses.

c) [2 pts] Individual masses.

d) [2 pts] Individual radii.

e) [2 pts] Ratio of the effective temperatures of the two stars.

Problem 4:

a) [4 pts] Compare the nucleosynthesis evolution of low-mass (stars like the sun) and high-mass (20 solar mass) stars. In particular, describe all of the hydrostatic and and/or explosive phases of element formation for each type of star. List the elements that are fused (or burned), the order that they happen during stellar evolution and the most likely products of those reactions.

b) [3 pts] What is the heaviest element that can be fused in low-mass and high-mass stars and why? What about iron fusion? When does it occur, or if not, why not? What about the heaviest elements such as precious metals? How are they formed? Describe the processes?

c) [3 pts] How do we know that nucleosynthesis occurs in stars? Give specific examples of observations that indicate element formation must occur in certain stars. What stage of evolution are these stars in, and how are the elements that we observe formed inside the star?

Problem 5:

A telescopic survey to find nearby "space rocks" can find moving objects to a magnitude of 18.5. The relationship between magnitude and flux for the "visible" passband used is:

$$mag = -2.5log(f/f_0)$$

where f is the flux from the target and f_0 is the flux from a mag = 0 object (assume $f_0 = 1.0E{-}8$ W m^{-2}).

An approximately spherical space rock, 50 meters in diameter, with an albedo of 0.2, comes near the Earth. The rock shines in the visible only by reflected sunlight.

a) [1 pts] Calculate the flux of sunlight in the visible at a distance of 1 AU from the Sun. Assume the "visible" pass band encompasses 1/3 of the bolometric power output of the Sun.

b) [2 pts] From the parameters given calculate the visible power of the rock (power of reflected sunlight) when approximately 1 AU from the Sun.

c) [4 pts] What is the maximum distance from Earth that the survey could detect the rock? (The rock will not emit isotropically, of course, but only from its illuminated side. Just assume it reflects uniformly from half its surface (the "day" side)). Don't worry about the changing solar flux with distance- just assume the rock is near 1 AU from Sun.

d) [3 pt] Assume the rock has a density of a typical rocky asteroid. Assume it hits the Earth with a speed equal to the escape speed of the Earth. How many megatons of energy would be released by the impact? (1 MT = 4.2E15 Joules).

Problem 6:

The equation of radiative transfer in spherical coordinates is:

$$\mu \frac{\partial I_\nu}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I_\nu}{\partial \mu} = -\chi_\nu I_\nu + \eta_\nu$$

a) [3pts] Show that the moment equations can be written:

$$\frac{1}{r^2}\frac{\partial}{\partial \tau_\nu}(r^2 H_\nu) = (J_\nu - S_\nu)$$

$$\frac{\partial K_\nu}{\partial \tau_\nu} + \frac{(J_\nu - 3K_\nu)}{(\chi_\nu r)} = H_\nu$$

b) [2pts] Introduce the Eddington factor $f_\nu = K_\nu / J_\nu$ and rewrite the moment equations in terms of it.

c) [2pts] Explain the problem with deriving a single second order equation for J_ν as is done in the plane-parallel case.

d) [3pts] Show that in fact with the sphericity factor:

$$ln(r^2 q_\nu) = \int_{r_c}^{r} [(3f_\nu - 1)/(r' f_\nu)] dr' + ln(r_c^2)$$

where r_c is the radius of the opaque core, the two moment equations can be combined to give:

$$\frac{\partial^2}{\partial X_\nu^2}(r^2 q_\nu f_\nu J_\nu) = q_\nu^{-1} r^2 (J_\nu - S_\nu)$$

where $dX_\nu = q_\nu d\tau_\nu$.

CONSTANTS

$$\sigma = 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4}; \quad c = 3.00 \times 10^{10} \text{ cm s}^{-1}; \quad T_{\odot} = 5,800\text{K}$$

$$G = 6.67 \times 10^{-8} \text{ g}^{-1} \text{ cm}^{3} \text{ s}^{-2}; \quad k = 1.38 \times 10^{-16} \text{ erg K}^{-1}$$

$$m_H = 1.67 \times 10^{-24} \text{ g}; \quad m_e = 9.11 \times 10^{-28} \text{ g}; \quad M_{\odot} = 1.99 \times 10^{33} \text{ g}$$

$$M_{\text{earth}} = 5.97 \times 10^{27} \text{ g}; \quad M_G = 4.0 \times 10^{11} M_{\odot}$$

$$h = 6.63 \times 10^{-27} \text{ erg s}; \quad a = 7.56 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}$$

$$R_{\odot} = 6.96 \times 10^{10} \text{ cm}; \quad R_{\text{earth}} = 6.37 \times 10^{8} \text{ cm}$$

$$1 \text{ AU} = 1.496 \times 10^{13} \text{ cm}$$

$$1 \text{ parsec} = 3.09 \times 10^{18} \text{ cm}; \quad 1\mathring{A} = 10^{-8} \text{ cm}$$

$$M_V(\odot) = 4.8; \quad M_{bol}(\odot) = 4.7; \quad L_{\odot} = 3.9 \times 10^{33} \text{ ergs s}^{-1}$$

$$1 \text{ year} = 3.16 \times 10^{7} \text{ s}$$