ASTRONOMY QUALIFYING EXAM

August 2010

Possibly Useful Quantities

$L_\odot$ = 3.9 x 10^{33} erg s^{-1}
$M_\odot$ = 2 x 10^{33} g
$M_{bol\odot} = 4.74$
$R_\odot$ = 7 x 10^{10} cm
1 A.U. = 1.5 x 10^{13} cm
1 pc = 3.26 l.y. = 3.1 x 10^{18} cm
a = 7.56 x 10^{-15} erg cm^{-3} K^{-4}
c= 3.0 x 10^{10} cm s^{-1}
σ = ac/4 = 5.7 x 10^{-5} erg cm^{-2} K^{-4} s^{-1}
k = 1.38 x 10^{-16} erg K^{-1}
e = 4.8 x 10^{-10} esu
1 fermi = 10^{-13} cm
N_A = 6.02 x 10^{23} moles g^{-1}
G = 6.67 x 10^{-8} g^{-1} cm^3 s^{-2}
m_e=9.1 x 10^{-28} g
h = 6.63 x 10^{-27} erg s
1 amu = $1.66053886 \times 10^{-24}$ g

PROBLEM 1

The orbit of the asteroid Ceres has a semimajor axis $a = 2.77\,\mathrm{AU}$. Assume that it is spherical with radius $R = 500\,\mathrm{km}$, and that its orbit can be approximated as circular.

a. (1 pt) What is the orbital period, in years?

b. (2 pts) Assume that Earth and Ceres orbit in the same plane and in the same direction. At opposition, what is the relative velocity, in km s^{-1}, of Ceres with respect to Earth?

c. (2 pts) From Earth, how fast would Ceres appear to be moving on the sky, in arcsec s^{-1}?

d. (2.5 pts) You observe Ceres at opposition with the OU telescope which has a focal length of 4 meters and an objective diameter of 0.4 meters. You use a CCD that measures 11.6 mm on a side. How long does it take for the asteroid to appear to move from one side of the CCD to the other?

e. (2.5 pts) The apparent visual magnitude of the sun is $m_V = -26.7$. If Ceres has an albedo of 0.5, what is its apparent visual magnitude when it is at opposition?

PROBLEM 2

Consider a spherical blackbody of constant temperature T and mass M, whose surface lies at coordinate $r = R$. An observer located at the surface of the sphere and a distant observer both measure the blackbody emission of the sphere.

a. (3 pts) If the observer at the surface of the sphere measures the luminosity of the blackbody to be L, use the gravitational time dilation formula

$$\frac{\Delta t_0}{\Delta t_\infty} = \frac{\nu_\infty}{\nu_0} = \left(1 - \frac{2GM}{r_0 c^2}\right)^{1/2}$$

to show that an observer at infinity measures

$$L_\infty = L(1 - \frac{2GM}{Rc^2})$$

b. (3 pts) Both observers use Wien's law,

$$\lambda_{\text{max}} T = 0.3 \text{ K cm}$$

to determine the blackbody's temperature. Show that

$$T_\infty = T\sqrt{1 - \frac{2GM}{Rc^2}}$$

c. (3 pts) Both observers use the Stefan-Boltzmann law,

$$L = 4\pi R^2 \sigma T^4$$

to determine the radius of the blackbody. Show that

$$R_\infty = \frac{R}{\sqrt{1 - \frac{2GM}{Rc^2}}}$$

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d. (1 pt) If we ignored GR effects, would we over– or underestimate the value of R?

PROBLEM 3

a. (3 pts) Using the luminosity equation for radiative transport

$$\frac{L}{4\pi r^2} = -\frac{4}{3}\frac{acT^3}{\kappa\rho}\frac{dT}{dr},$$

use dimensional analysis to find an expression for the luminosity in terms of the mass, such that L $\propto M^{\alpha}$. Assume that the opacity, κ, is entirely due to electron scattering and is independent of mass or density.

b. (3 pts) Now assume that the actual luminosity of the star is

$$L = 4 \times 10^{33}(M/M_{\odot})^{\alpha} \text{ ergs/sec.}$$

Using the α you found in part (a) and assuming blackbody emission, calculate the effective temperature of a star that has M = 3 $M_{\odot}$ and R = 2 $R_{\odot}$. For this temperature, use the Saha equation

$$log\Big[\frac{N_{i+1}}{N_i}\Big] = 2.5logT - \frac{5040}{T}\chi_i - 0.18$$

(where χ_i for H is 13.6 ev and $logP_e = 0$ has been used) to determine the fraction of total H atoms that are ionized in the atmosphere of this star. Do you expect that such a star will have strong or weak H spectral lines, and why? What is the spectral type of this star?

c. (4 pts) Calculate and compare the free-fall, Kelvin-Helmholtz and nuclear time scales (in years) for this star.

The free–fall acceleration is given by

$$\frac{|d^2R|}{|dt^2|} = g.$$

Use dimensional analysis to get an expression for the free–fall time scale, τ_{f-f}, in terms of the average density $\bar{\rho}$. For our star of M = 3 $M_{\odot}$ and R = 2 $R_{\odot}$, calculate τ_{f-f}.

For the Kelvin–Helmholtz time scale, use the luminosity in part (b) and the total gravitational potential energy available to this 3 $M_{\odot}$ star.

For the nuclear time scale assume that only 10% of the star's mass contributes to energy generation and the luminosity is given in (b). (Assume 0.7 % mass loss in the nuclear conversion.)

In terms of stellar evolution do your numbers for these time scales make sense? If not, why not?

PROBLEM 4

Assume a two level atom with one bound electron. The atom is not hydrogen or helium.

a. (2 pts) List and explain all possible ways to excite and de-excite the electron.

b. (2 pts) Which processes are irrelevant in the case of a diffuse nebula such as an H II region? Why?

c. (2 pts) If N_1, N_2, and N_e are number densities of atomic levels 1, 2, and free electrons, respectively, and q_{12}, q_{21}, and A_{21} are the collisional excitation, de-excitation, and spontaneous rate coefficients between levels 1 and 2, derive an expression for N_2/N_1 in terms of the other quantities.

d. (2 pts) Using the results of part c., write down an expression for the energy emitted in a forbidden line per second per cm^3 in terms of N_1, N_e, q_{12}, q_{21}, and A_{21}. Explain why the production rate increase with density of this radiation slows as the density rises.

e. (2 pts) Explain the difference between a permitted transition and a forbidden transition using concepts of quantum mechanics. Why are forbidden lines more likely to be observed in low density regimes such as H II regions than in high density regimes such as broad line regions of AGNs? Refer to your results in part d.

PROBLEM 5

This question is about the broadening of spectral lines of neutral iron, Fe I, in the context of a 1D plane–parallel solar atmosphere.

a. [3 pts] Name and discuss two kinds of line–broadening profiles that are treated as Lorentzian, and two kinds that are treated as Gaussian.

b. [3 pts] What is a Voigt profile? Sketch one, and discuss it in terms of Lorentzians and Gaussians.

c. [4 pts] Sketch a curve of growth [$log(W_\lambda)$ versus $log(N_\ell f)$, where W_λ is the equivalent width, N_ℓ is the number of absorbers per unit volume in the lower level of the transition, and f is the oscillator strength]. For each segment of the curve, give an approximate expression for the dependence of W_λ on $N_\ell f$, on the Gaussian broadening parameter $\Delta\lambda_D$, and on the Lorentzian broadening parameter Γ.

PROBLEM 6

a. (4 pts) Use conservation of energy and momentum to derive the dependence of matter density and radiation density on redshift, z, and equation of state, $w = p/\rho$.

b. (1 pt) What can you say about the radiation density compared with the matter density at the current epoch of the universe?

c. (4 pts) Assuming a flat universe, what is the expansion rate of the universe as a function of redshift and the matter, radiation, and dark–energy densities?

d. (1 pt) From observational data, what do we know about how the cosmic expansion rate is changing at the current epoch?