Consider Nova Scorpii 94. Observations indicate a binary with a putative black hole and a subgiant star.
a. (4 points) First decscribe how we know it is a binary and how it is predicted that the unseen companion in this binary is suspected to be a black hole. What are the assumptions?
b (6 points) An important discovery in this binary was the presence of alpha-elements such as neon, oxygen, silicon in the observed subgiant star. These elements were in proportions far above solar and could not have been produced in the observed subgiant star. These elements are only produced in explosive burning nucleosynthesis at temperatures of about 1 billion degrees. What does this suggest about the final evolution of the star that became a black hole? Describe the scenario. In other words describe how this black hole formed. How might we test that scenario?
a. (4 pts) Evalute the energy of the blackbody photons inside your eye. Take your eye to be a hollow sphere of radius 1.5 cm .
b. (4 pts) Compare the result in (a.) with the visible energy inside your eye while looking at a 100 -Watt light bulb that is 1 meter away. You may assume that the light bulb is $100 \%$ efficient, although in reality it converts only a few percent of its 100 watts into visible photons. The area of the eye's pupil is about $0.1 \mathrm{~cm}^{2}$.
c. ( 2 pts ) Why is it dark when you close your eyes?

The surface temperature of an asteroid or other body with negligible internal heat orbiting the Sun is determined by the sunlight the body absorbs and the radiation the body emits. The amount of sunlight absorbed depends on object's albedo (A), distance from Sun (d) and size.
a. (4 pts) Assume we have a spherical asteroid that is spinning rapidly and chaotically so that it has a uniform surface temperature. Derive an equation that relates the surface temperature of the asteroid $\left(\mathrm{T}_{\text {ast }}\right)$ to the relevant parameters of the Sun and asteroid.
b. (1 pt) For asteroids in the main asteroid belt, give a range of A values usually encountered. What is the size of the largest main belt asteroid?
c. $(2 \mathrm{pts})$ Calculate the temperature for an asteroid with an albedo at the lower end of range you gave in b), located at a distance from the Sun equal to the distance where the main belt asteroid distribution peaks.
d. ( 1 pt ) Pluto has an albedo of 0.7 . Why does it have such a high albedo?
e. (2 pt) Pluto has a semimajor orbital axis of 40 AU , and an orbital eccentricity of 0.25 . Calculate the minimum and maximum temperature of Pluto over the course of one of its orbits around the Sun. Assume Pluto has a uniform temperature over its entire surface.
a. (3 pts) Contrast Pop. I and Pop. II stars in the Milky Way Galaxy in terms of their general location, kinematics, and metallicity.
b. (2 pts) Give a plausible model for the formation of the Milky Way which explains the differences discussed in part a).
c. (2 pts) According to chemical evolution theory, why does the value of $[\mathrm{Fe} / \mathrm{O}]$ in any one location in the Galaxy tend to increase with time? If the initial mass function were flatter (higher fraction of massive stars), how would you expect that to affect the evolution of the local value of $[\mathrm{Fe} / \mathrm{O}]$ ? Explain.
d. (3 pts) Using simple physics and assuming circular orbits for stars in the disk, derive a functional relation between surface density $\sigma$ (mass $/ \mathrm{pc}^{2}$ ), tangential (circular) velocity $v$, and galactocentric distance $r$. Show your work. What is implied about the behavior of surface density in regions of the Milky Way disk where the rotation curve is flat? Explain.

## PROBLEM 5

For the case of plane parallel radiative transfer:
a. (3 pts) Write down the equation of transfer with the coordinate $z$ as the independent variable and using the emissivity. Define all the terms and give their units.
b. (2 pts) Define the optical depth $\tau_{\nu}$ and rewrite the equation of transfer in terms of $\tau_{\nu}$. Again, define all the terms and give their units.
c. (3 pts) Consider a finite slab $\tau_{1} \leq \tau_{\nu} \leq \tau_{2}$. Write down the formal solution for $I_{\nu}\left(\tau_{1}, \mu\right)$ in terms of $I_{\nu}\left(\tau_{2}, \mu\right)$.
d. (2 pts) Explain why the answer in part (c) is only the formal solution and not just the solution.

## PROBLEM 6

The measurement of distance-redshift relations is fundamental in observational cosmology, as these relations probe the cosmological model that describes our universe.
a. ( 3 pts ) Write down the expansion rate of the universe as a function of redshift depending on the cosmological parameters $H_{0}, \Omega_{m}, \Omega_{r}, \Omega_{k}$, and $\Omega_{\Lambda}$.
b. (2 pts) Write down the Robertson-Walker metric for a spherically symmetric universe with zero spatial curvature.
c. (3 pts) Use your answers to (a) and (b) to derive the comoving distance $r(z)$ from the observer to an object at redshift $z$.
d. (2 pts) Express $r(z)$ to second order in $z$ in the limit of $z \ll 1$

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\(\mathrm{e}=4.8 \times 10^{-10} \mathrm{esu}\)
1 fermi \(=10^{-13} \mathrm{~cm}\)
\(\mathrm{L}_{\odot}=3.9 \times 10^{33} \mathrm{ergs} / \mathrm{sec}\)
\(\mathrm{M}_{\odot}=2 \times 10^{33} \mathrm{gm}\)
\(\mathrm{a}=7.56 \times 10^{-15} \mathrm{erg} \mathrm{cm}^{-3} \mathrm{deg}^{-4}\)
\(\mathrm{c}=3.0 \times 10^{10} \mathrm{~cm} / \mathrm{sec}\)
\(\mathrm{k}=1.38 \times 10^{-16} \mathrm{erg} / \mathrm{deg}\)
\(\mathrm{R}_{\odot}=7 \times 10^{10} \mathrm{~cm}\)
1 year \(=3.16 \times 10^{7}\) seconds,
\(\mathrm{N}_{A}=6.02 \times 10^{23} \mathrm{moles} / \mathrm{gm}\)
\(\mathrm{G}=6.67 \times 10^{-8} \mathrm{gm}^{-1} \mathrm{~cm}^{3} \mathrm{~s}^{-2}\)
\(\mathrm{m}_{e}=9.1 \times 10^{-28} \mathrm{gm}\)
\(\mathrm{h}=6.63 \times 10^{-27} \mathrm{erg} \mathrm{sec}\)
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