Astronomy Qualifier - August 2007

Lots of necessary (and some unnecessary) "constants" and possibly useful integrals at end.

Problem 1:

1. Imagine a pure hydrogen cloud surrounding a hot star with T_{eff}=42,000K and a radius of 12 $R_\odot$ which radiates as a blackbody. Before the star began to radiate, the density of the cloud was 100 atoms per cm^{-3}.

a) [5 pts] Using the Wien approximation, compute the stellar luminosity of ionizing photons.

b) [2 pts] If the recombination rate coefficient, α, is 2.59×10^{-13} cm^3/s, what is the Strömgren radius of the cloud in parsecs?

c) [2 pts] Explain in words how the emission lines of Hα and Hβ from this cloud can be used to estimate the amount of interstellar reddening taking place along the line of sight to the cloud.

d) [1 pts] If this cloud is 1 kpc from the sun, what is its angular diameter in arcseconds?

Hint: The Wien approximation for blackbody radiation is:

$$\pi B_\nu = 2\pi h c^{-2} \nu^3 e^{-h\nu/kT} \tag{1}$$

Problem 2:

$$m_{Si^{28}} = 27.977\text{amu}$$

$$m_{Fe^{56}} = 55.935\text{amu}$$

Calculate the efficiency of the reaction

$$Si^{28} + Si^{28} \longrightarrow Fe^{56}$$

a) [2 pts] Assume that in a 15$M_\odot$ star, the Si burning core has a mass 1.1$M_\odot$. Calculate the total energy available for Si burning.

b) [2 pts] Assume that the luminosity during the Si burning stage is 3.4×10^{44} ergs s^{-1} and calculate the lifetime for Si burning.

c) [1 pt] What form is this luminosity in?

d) [2 pts] The iron core in the center of a massive star can be considered an "iron white dwarf". The mass of a white dwarf is

$$5.76 Y_e^2 \mathrm{M}_\odot$$

Calculate the maximum mass of an iron white dwarf.

e) [1 pt] Calculate the maximum mass of a C+O white dwarf.

f) [2 pts] The actual Chandrasekhar mass of a C+O white dwarf is 1.39 $M_\odot$. Explain why it is different than the answer you found above.

Problem 3:

a) [2 pts] What are the six most abundant elements at the photosphere of the sun? To the extent that you can (perfection is not required) list them in order of decreasing abundance.

b) [1 pt] Do the same for the center of the sun. Briefly discuss the processes responsible for any differences from your answer for the photosphere.

c) [2 pts] Do the same for the center of a 10 solar–mass star that is about to leave the main sequence. Briefly discuss the processes ...

d) [2 pts] What will be the two most abundant elements in the white dwarf that the sun will eventually become? Explain your answer.

e) [3 pts] The dominant spectral features of cool giant stars are particularly sensitive to the abundance ratio of which two elements? Explain why, and mention the spectral features that are most prominent when the ratio is less than unity, and when it exceeds unity.

Problem 4:

a) [3 pts] Using the equation for hydrostatic equilibrium and dimensional analysis, write down an expression for pressure, P, in terms of the mass, radius and any constants. Now evaluate this expression for the Sun. In other words, determine the central pressure for the Sun.

b) [3 pts] Using your result in a) and the equation of state for an ideal gas, in terms of density and temperature, derive an expression for the central temperature of the Sun. Assuming 100% ionized H gas and that the mass density is 3 times the average density, determine numerically the central temperature of the Sun. (Note: you do not need to know or calculate the average density; just assume that the average density is given by the standard approximation relating volume, M and R.)

c) [4 pts] What is the virial theorem? Using the expression for gravitational potential energy, write down an expression (in terms of the solar luminosity) for the time scale over which gravitational contraction could have supplied the luminosity radiated by the Sun at its current rate. What is the name for this time scale? Numerically determine this time scale (in years) for the Sun. How does this compare with the nuclear time scale, assuming 0.7% mass loss over the entire solar mass?

Problem 5:

The time rate of change of the deuterium abundance in the PP(proton-proton) cycle is given by

$$dD/dt = -N_D N_H < \sigma v >_{HD} +1/2 N_H^2 < \sigma v >_{HH} .$$

a) [4 pts] Calculate the equilibrium ratio of the number density of deuterium, N_D, to the number density of hydrogen, N_H for a temperature of T_6= 10. The reaction rate for the HD reaction is given by

$$< \sigma v >_{HD} = \frac{7.2x10^{-19}}{A} S_0 \tau^2 e^{-\tau}$$

with S_0 = 2.5 x 10^{-4} kev-barns, A is the reduced atomic weight = $\frac{A_1 A_2}{(A_1+A_2)}$ and

$$\tau = 42.48(Z_1^2 Z_2^2 A/T_6)^{1/3}$$

S_0 for the HH reaction, $< \sigma v >$, is 3.78 x 10^{-22}. How does your answer compare to the terrestrial value of $N_D/N_H \sim 10^{-4}$? Explain this discrepancy. Could the Sun have produced this terrestrial deuterium?

b) [3 pts] What are the approximate energy dependencies ($\epsilon \propto T^n$) of the PP Cycle, the CNO cycle and the 3α process? Explain why they are so different. Describe how and why energy is transported throughout various parts of a massive star and a low-mass star, like the Sun, while they are on the main sequence. (Draw some pictures!)

c) [3 pts] Describe the major nuclear burning phases (what is burned, what are the products, approximate temperatures and timescales) that occur in low mass and high mass stars. What are the TWO reasons why iron fusion does not occur? How are the elements heavier than iron produced in stars? What is the evidence for nucleosynthesis in stars ?

Problem 6:

The ability of a planet to retain an atmosphere depends on the temperature of the planet, its escape speed, and the mass of the individual gas particles. As a crude approximation, assume that a planet will retain an atmosphere if the planetary escape speed is 10 times or more the *rms* speed of the gas particles. Assume a planet with Earth- like density (5.5 times that of water) with an atmospheric temperature of 250 Kelvin. The planet has a primordial atmosphere of CO_2 (carbon dioxide) and molecular hydrogen (H_2).

a) [4 pts] What is the minimum radius of the planet that can retain an atmosphere of CO_2?

b) [2 pts] What is the minimum radius of the planet that can retain an atmosphere of H_2?

c) [2 pts] Write a description of the atmospheres of the 5 terrestrial bodies in our solar system (4 inner planets plus Luna). You should include the main species of gas particles in the atmosphere, and some indication of the surface density and/or pressure of the atmosphere (say in terms of the parameters of the gas in this room). Include a brief discussion of why the chemical composition of these atmospheres is so very different from cosmic abundances.

d) [2 pts] In reality, of course, the retention of atmospheres is much more complicated than the simple calculation above. Explain why both Venus and Mars appear to have much lower atmospheric abundances of water vapor than does Earth.

CONSTANTS

$$\sigma = 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4}; \quad c = 3.00 \times 10^{10} \text{ cm s}^{-1}; \quad T_\odot = 5,800\text{K}$$

$$G = 6.67 \times 10^{-8} \text{ g}^{-1} \text{ cm}^3 \text{ s}^{-2}; \quad k = 1.38 \times 10^{-16} \text{ erg K}^{-1}$$

$$m_H = 1.67 \times 10^{-24} \text{ g}; \quad m_e = 9.11 \times 10^{-28} \text{ g}; \quad M_\odot = 1.99 \times 10^{33} \text{ g}$$

$$M_{\text{earth}} = 5.97 \times 10^{27} \text{ g}; \quad M_G = 4.0 \times 10^{11} M_\odot$$

$$h = 6.63 \times 10^{-27} \text{ erg s}; \quad a = 7.56 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}$$

$$R_\odot = 6.96 \times 10^{10} \text{ cm}; \quad R_{\text{earth}} = 6.37 \times 10^{8} \text{ cm}$$

$$1 \text{ AU} = 1.496 \times 10^{13} \text{ cm}$$

$$1 \text{ parsec} = 3.09 \times 10^{18} \text{ cm}; \quad 1\mathring{A} = 10^{-8} \text{ cm}$$

$$M_V(\odot) = 4.8; \quad M_{bol}(\odot) = 4.7; \quad L_\odot = 3.9 \times 10^{33} \text{ ergs s}^{-1}$$

$$1 \text{ year} = 3.16 \times 10^{7} \text{ s}$$