ASTRONOMY QUALIFYING EXAM
August 2017

Possibly Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
a = 7.56 $\times 10^{-15}$ erg cm^{-3} K^{-4}
c = 3 $\times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
k = 1.38 $\times 10^{-16}$ erg K^{-1}
e = 4.8 $\times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
G = 6.67 $\times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s
1 amu = 1.66053886 $\times 10^{-24}$ g

PROBLEM 1

a) (3 points) Calculate the orbital semi-major axis ($a_{\odot}$) of the Suns orbit about the barycenter of the Solar System, in AU, in response to Jupiters orbital motion. Since Jupiter constitutes ~70% of the non-solar mass of our Solar System, you can ignore Solar System bodies less massive than Jupiter in your computation. Assume $a_{\rm Jupiter} = 5.2$ AU.

b) (2 points) To an external observer, what would be the transit depth of an Earth-size planet located at a=0.1AU (assume circular orbit) about an M dwarf star (Mass = $0.3M_{\odot}$; Radius = $0.8\ R_{\odot}$)?

c) (3 points) To an external observer, what would be the transit duration (in hours) of an Earth-size planet located at a=0.1AU (assume circular orbit) about a M dwarf star (Mass = $0.3\ _{\odot}$; Radius = $0.8\ R_{\odot}$)?

d) (2 points) To an external observer located 20 pc away, what would be the angular separation in arcseconds between an Earth-size planet located at a=0.1 AU and its host star?

PROBLEM 2

Consider a spherically symmetric main sequence (MS) star.
a) (1 point) Consider a spherical shell at radial position $(R, R + dr)$. Draw all the forces on it.
b) (2 points) From the above picture derive the equation of hydrostatic equilibrium (HSE).
c) (1 point) Assuming you wrote the HSE using R as independent variable, rewrite it using the enclosed mass M(R) as independent variable.
d) (2 points) Now finish off the equations of stellar structure by remembering we need 2 conservation laws and a way to relate pressure to other quantities. You may use either R or M as independent variable and you are given that ϵ is the energy generation rate per gram.
e) (2 points) OK now we have dealt with everything but the transport of energy. How is energy transported in MS stars?
f) (1 point) What energy transport process is *not* important in MS stars?
g) (1 point) What type of stars is it (the process from f) important in and why?

PROBLEM 3

In an attempt to identify the important components of AGB mass loss, various researchers have proposed parameterization of the mass loss rate that are based on fitting observed rates for a specified set of stars with some general equation that includes measurable quantities associated with the stars in a sample. One of the most popular, developed by D. Reimers is given by

$$\dot{M} = -4 \times 10^{-13} \eta \frac{L}{gR} M_\odot \mathrm{yr}^{-1},$$

where L, g, and R are the luminosity, surface gravity, and radius of the star respectively, all in solar units (i.e., L is the luminosity in terms the luminosity of the sun, etc.), and η is a free parameter whose value is expected to be near unity.

a) (1 point) Explain qualitatively why L, g, and R enter the equation the way they do.

b) (2 points) Estimate the mass loss rate of a 1 $M_\odot$ AGB star that has a luminosity of 7000 $L_\odot$ and a temperature of 3000K. Assume $\eta = 1$.

c) (2 points) Show that the Reimer's mass loss rate can also be written in the form

$$\dot{M} = -4 \times 10^{-13} \eta \frac{LR}{M} M_\odot \mathrm{yr}^{-1},$$

where, as before, L, R and M are all in solar units.

d) (3 points) Assuming (incorrectly) that L, R, and η do not change with time, derive an expression for the mass of the star as a function of time. Assume also that $\eta = 1$. Let $M = M_0$ when the mass loss phase begins.

e) (2 points) How long would it take for a star with an initial mass of $1\,M_\odot$ to be reduced to the mass of the degenerate carbon-oxygen core (0.6 $M_\odot$)?

PROBLEM 4

The APO 3.5m telescope has a cross-dispersed echelle spectrograph that provides a resolution of ∼31,500 at optical wavelengths. The echelle grating has 31.6 grooves/mm and an incident angle of 69.5 degrees.

a) (1 point) Diffraction gratings disperse light across multiple orders as given by the grating equation

$$\mathsf{m}\lambda = \mathsf{d}(\sin(\theta_\mathsf{i}) + \sin(\theta_\mathsf{m})) \tag{1}$$

where m is the order, d is the distance between grooves, $\theta_{\rm i}$ is the incident angle, and $\theta_{\rm m}$ is the diffracted angle. For a fixed incident angle, it is clear that the diffracted angle depends upon both wavelength and order such that light from adjacent orders can overlap (i.e., equivalent $\theta_{\rm m}$). What is the width of the spectrum that is not contaminated by adjacent orders for order 55, centered at 5175 Angstroms? This is also known as the free spectral range (FSR).

b) (1 point) What is the function of a "cross disperser"?

c) (1 point) How many detector pixels are needed to observe the full FSR with 2.5 pixels per resolution element? Why do we want 2.5 pixels per resolution element?

d) (3 points) What type of calibrations need to be taken for optical spectroscopy when radial velocity measurements are desired? Define the purpose of each calibration.

e) (4 points) Outline an observing plan to conduct optical radial velocity observations of a star. Starting in the afternoon to the full observation of a science target.

PROBLEM 5

a) (2 points) Consider a planetesimal with radius, s, and mass, m, is embedded in a sea of identical planetesimals. These planetesimals have a typical velocity relative to relative to one another of σ, and they have a volume density of ρ. **Ignoring any gravitational focusing**, calculate the rate $\frac{\mathrm{dm}}{\mathrm{dt}}$ that a planetesimal's mass will grow as it collides with others. Assume an ideal situation where two planetesimals will stick together if they come within s of one other.

b) (1 point) Now consider the close encounter of two planetesimals, accounting for their gravity. Assume that during the close encounter they attain a maximum relative velocity of $\mathrm{v_{max}}$ and a minimum distance of $\mathrm{d_{min}}$. First write down the initial total energy of the two planetesimals before they begin their close encounter. (Assume they are initially separated by a huge distance.)

c) (1 point) Now write down the energy of the two planetesimals at their point of closest approach in terms of $\mathrm{v_{max}}$ and $\mathrm{d_{min}}$.

d) (1 point) Assume the two planetesimals began approaching each other with an initial impact parameter of b. Write down the initial angular momentum of the system.

e) (1 point) Write down the angular momentum of the system at the point of closest approach in terms of $\mathrm{v_{max}}$ and $\mathrm{d_{min}}$.

f) (1 point) Using your expressions from parts d and e, solve for $\mathrm{v_{max}}$ in terms of $\mathrm{d_{min}}$ and b.

g) (2 points) Substitute the expression from part f expression into part c, and use conservation of energy to determine the largest impact parameter, $\mathrm{b_{max}}$, that will result in a collision between two planetesimals (i.e. $\mathrm{d_{min}} \leq \mathrm{s}$).

h) (1 point) Using your expressions from parts a and g, what is the ratio of the gravitational cross-section to the physical cross-section of a planetesimal?

PROBLEM 6

Consider a spherical shell of radius r and mass m in a Universe filled with nothing but pressureless dust. Assume that the total energy of the shell is given by

$$E = -\frac{1}{2} m k c^2 \varpi^2, \tag{2}$$

where k is a constant and ϖ is a comoving coordinate such that $r(t_0) = \varpi$ and $r(t) = R(t)\varpi$. R(t) is the dimensionless scale factor.

a) (3 points) Show that

$$v^2 - \frac{8}{3}\pi G \rho r^2 = -k c^2 \varpi^2 \tag{3}$$

b) (1 points) Discuss the fate of this Universe for $k < 0, k = 0, k > 0$.

c) (3 points) Show that the expansion of this model Universe can be described by

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G \rho\right] R^2 = -k c^2 \tag{4}$$

d) (3 points) Estimate the critical density (as a function of the Hubble parameter) for a Flat Universe.