

# E & M Qualifier

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August 16, 2017

**To insure that the your work is graded correctly you MUST:**

1. use only the blank answer paper provided,
2. use only the reference material supplied (Schaum's Guides),
3. write only on one side of the page,
4. start each problem by stating your units e.g., SI or Gaussian,
5. put your alias (**NOT YOUR REAL NAME**) on every page,
6. when you complete a problem put 3 numbers on **every** page used for **that** problem as follows:
  - (a) the first number is the problem number,
  - (b) the second number is the page number for **that** problem (start each problem with page number 1),
  - (c) the third number is the total number of pages you used to answer **that** problem,
7. **DO NOT** staple your exam when done.

# Problem 1: Electrostatics

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A non-conducting solid sphere of radius  $R$  carries a charge density  $\rho(r) = kr$  (where  $k$  is a constant).

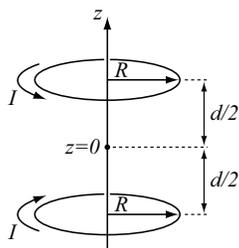
- (a) Find the electric field at a distance  $r$  such that  $r \geq R$  [1 point]
- (b) Find the electric field at a distance  $r$  such that  $r \leq R$  [1 point]
- (c) State the boundary conditions on the electric field components on the surface of the sphere, and show that your answers to parts a) and b) are consistent with them.  
HINT: The surface charge density of the sphere is zero in this case. [1 point]
- (d) Calculate the electric potential for all  $r$  using  $\lim_{r \rightarrow \infty} V(r) = 0$  [2 points]
- (e) Find the work required to assemble this charge [2 points]
- (f) If the *non-conducting* solid sphere was replaced by a *conducting* solid sphere with the same total charge, how does that change your answer to parts a, b, and c?  
Explicitly show that the new field satisfies the new boundary conditions across the surface of the sphere. [3 point]

## Problem 2: Magnetostatics

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Consider a circular loop of radius  $R$  which carries a steady current  $I$ .

- (a) Calculate the magnetic field a distance  $z$  above the center of the loop. [3 points]
- (b) Now consider a configuration composed of two circular loops a distance  $d$  apart and with currents flowing in opposite directions, as shown in the figure. This configuration is known as anti-Helmholtz coils. Calculate the magnetic field along the  $z$ -axis as a function of  $z$ . [2 points]



- (c) For what value of  $z$  will the magnetic field due to the anti-Helmholtz coils be equal to zero? Give a physical explanation for your result. [2 points]
- (d) Calculate the magnetic dipole moment of the configuration. [3 points]

## Problem 3: Waves

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An electromagnetic wave with an angular frequency of  $\omega$  passes from medium 1, through a slab of medium 2 (with thickness  $d$ ), and into medium 3. All three media are linear and homogeneous, and have a permeability of  $\mu_o$ . The index of refraction is  $n_1$ ,  $n_2$ , and  $n_3$  for medium 1, 2, and 3, respectively.

- (a) In medium 1, there is an incident plane wave (electric field of amplitude  $E_I$ ) and a reflected plane wave (electric field of amplitude  $E_R$ ). In medium 3, there is a transmitted plane wave (electric field of amplitude  $E_T$ ). In medium 2, there is a plane wave going toward medium 3 (electric field of amplitude  $E_r$ ) and a plane wave going toward medium 1 (electric field of amplitude  $E_l$ ). Write down expressions that describe the electric and magnetic fields in medium 1, medium 2, and medium 3. [4 points]
- (b) Apply the boundary conditions for the electric and magnetic fields at the interface between medium 1 and medium 2. Express  $E_I$  in terms of  $E_r$ ,  $E_l$ ,  $n_1$ , and  $n_2$ . [1.5 points]
- (c) Apply the boundary conditions for the electric and magnetic fields at the interface between medium 2 and medium 3. Express  $E_r$  in terms of  $E_T$ ,  $n_2$ ,  $n_3$  and  $d$ . Also express  $E_l$  in terms of  $E_T$ ,  $n_2$ ,  $n_3$  and  $d$ . [1.5 points]
- (d) Combine your answers for part *b* and *c* to get an expression for  $E_T/E_I$  in terms of  $n_1$ ,  $n_2$ ,  $n_3$ , and  $d$ . What percentage of the incident wave is transmitted into medium 3? Express your answer in terms of  $n_1$ ,  $n_2$ ,  $n_3$  and  $d$ . [1 points]
- (e) Suppose medium 1 is water ( $n_1 = 4/3$ ), medium 2 is glass ( $n_2 = 3/2$ ), and medium 3 is air ( $n_3 = 1$ ). Will the thickness of the glass make much difference in how well you (in medium 3) can see the fish (in medium 1)? How would your answer change if instead you were in the water and the fish was in the air? [2 points]

## Problem 4: ED in media

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In this problem we consider the propagation of an electromagnetic wave through an optically active media; a media that causes the direction of polarization to rotate about the direction of propagation. Such a media can be describe using the susceptibility tensor

$$\boldsymbol{\chi} = \begin{pmatrix} \chi_{11} & i\chi_{12} & 0 \\ -i\chi_{12} & \chi_{11} & 0 \\ 0 & 0 & \chi_{33} \end{pmatrix}.$$

Furthermore, recall that the polarization of the media and the electric field are related by  $\vec{P} = \epsilon_0 \boldsymbol{\chi} \vec{E}$  in SI units. *You can assume a non-magnetic linear dielectric material with no free charges or currents.*

- (a) Starting from the Maxwell equations, the expressions given in the statement of the problem, and assuming a transverse electromagnetic wave propagating in the  $z$  direction, derive the magnitude of the wave vector  $k$  for the two possible polarizations in terms of the components of  $\boldsymbol{\chi}$ . [4 points]
- (b) Prove that the allowed wave modes correspond to circularly polarized waves. [2 points]
- (c) Under the assumption that  $\chi_{12} \ll \chi_{11}$ , derive an expression for the amount that the polarization rotates over a distance  $\ell$ . The approximation  $\sqrt{1 - \epsilon} \approx 1 + \epsilon/2$  might be useful. [4 points]

## Problem 5: Radiation

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Two oscillating dipole moments  $\vec{d}_1$  and  $\vec{d}_2$  are oriented parallel to each other in the direction of the  $y$ -axis and are separated by a distance  $L$ . They oscillate in phase at the same frequency  $\omega$ . For an observer at a distance  $r$  with  $r \gg L$  located at an angle  $\theta$  with respect to the  $y$ -axis in the plane of the two dipoles.

(a) Show that [9 points]

$$\frac{dP}{d\Omega} = \frac{\omega^4 \sin^2 \theta}{8\pi c^3} (d_1^2 + 2d_1 d_2 \cos \delta + d_2^2), \quad (1)$$

where

$$\delta = \frac{\omega L \sin \theta}{c} \quad (2)$$

and where  $P$  is the time-averaged power. The units are Gaussian.

(b) Show that when  $L \ll \lambda$ , the radiation can be approximated as from a single oscillating dipole of amplitude  $d_1 + d_2$ . [1 point]

## Problem 6: Relativity

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- (a) Write down both the homogeneous and inhomogeneous Maxwell's equations in manifestly Lorentz covariant form using the 2nd rank field strength tensor  $F_{\mu\nu}$  and its dual  $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ . (State which system of units you are using) [2 points]
- (b) Write down the components (in matrix form) of  $F_{\mu\nu}$ . [2 points]
- (c) Lorentz transformations on a four-vector are given by  $x'_\mu = \Lambda_\mu^\nu x_\nu$ . Write down the form of  $\Lambda_\nu^\mu$  for a boost to velocity  $\beta_x = v_x/c$  along the  $x$ -direction. [2 points]
- (d) Using  $\Lambda_\nu^\mu$ , calculate the Lorentz transformation relations for  $\vec{E}$  and  $\vec{B}$  for a boost along the  $x$ -direction. [2 points]
- (e) Depict the lines of electric field  $\vec{E}$  from a point charge a) at rest and b) moving with some large velocity  $\beta_x$ . [2 points]