ASTRONOMY QUALIFYING EXAM
January 2020

Notes and Instructions

- There are 6 problems. Attempt them all as partial credit will be given.
- Write on only one side of the paper for your solutions.
- Write your alias on the top of every page of your solutions.
- Number each page of your solution with the problem number and page number (e.g. Problem 3, p. 2/4 is the second of four pages for the solution to problem 3.)
- You must show your work to receive full credit.

Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
1 radian = 206265 arcsec
a = 7.56 $\times 10^{-15}$ erg cm^{-3} K^{-4}
c = 3 $\times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
k = 1.38 $\times 10^{-16}$ erg K^{-1} = 8.6173×10^{-5} eV K^{-1}
e = 4.8 $\times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
G = 6.67 $\times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s = 4.1357 $\times 10^{-15}$ eV s
1 amu = 1.66053886 $\times 10^{-24}$ g

PROBLEM 1

a) [4 pts] Consider a cluster of 10,000 stars, including 100 stars with $M_V = 0$ mag and the rest with $M_V = 5$ mag. What is the"integrated" absolute V-band magnitude of the cluster?

b) [3 pts] If the cluster is 3 kiloparsecs away, what is its apparent magnitude?

c) [3 pts] If we observed a more "normal" cluster and found that the brightest main sequence stars in the cluster had an absolute magnitude of $M_V \simeq 5$, what would you estimate the age of this cluster to be? Explain your reasoning.

PROBLEM 2

Using the Virial theorem for a non relativistic, ideal gas, we find

$$\mathsf{E}_{\text{total}} = \mathsf{E}_{\text{int}} + \mathsf{E}_{\text{grav}} = -\mathsf{E}_{\text{int}} \tag{1}$$

therefore the star is bound.

a) [7 pts] Derive the same relationship for a classical relativistic gas of particles. Recall that the equation of state of a relativistic gas is $P = \frac{1}{3}U_{\text{int}}$, where U is the energy density.

b) [3 pts] Using your results from part a, estimate the total energy of the stars dominated by radiation pressure. Are such stars bound? Comment on the stability of such stars.

PROBLEM 3

In the center of an active galactic nucleus there exists a $10^8 M_\odot$ black hole. Material is fed from the surrounding region at a rate of $\frac{dM}{dt} \sim 1 M_\odot$ per year. This forms an accretion disk around the black hole. The high energy radiation emitted by the accretion disk is time variable and changes in luminosity on the ~ 1 hour timescale.

a) [2 pts] Calculate the Schwarzchild radius of this black hole.

b) [1 pt] Assume the luminosity variation is related to the size of the accretion disk (and neglect any general relativistic effects). Estimate the radius of the outer edge of the accretion disk from the photon time of flight.

c) [4 pts] Calculate the luminosity of the accretion disk and the accretion disk peak temperature given the mass infall rate and the calculated outer disk radius.

d) [3 pts] Assume the photon emission of the disk is blackbody. Calculate the peak wavelength of the photons emitted at the disk temperature.

PROBLEM 4

Define/explain 10 of the following and indicate their relevance in astronomy. Be sure to clearly indicate which of the 10 you would like graded (otherwise the first 10 items will automatically be graded).

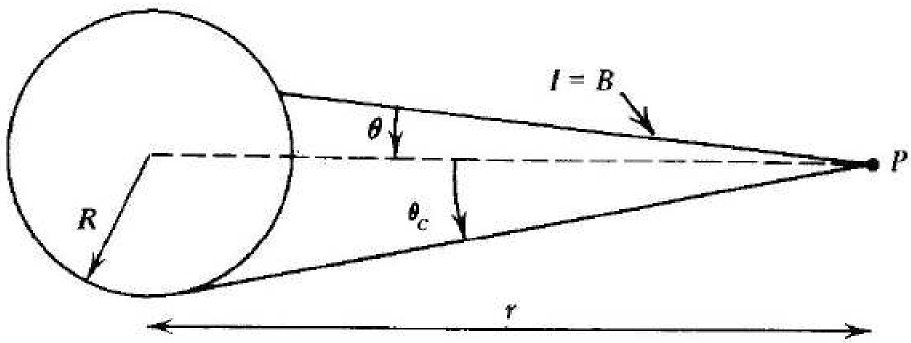
a) Initial Mass Function
b) Baade's Window
c) Extinction & reddening
d) Flat field calibrations
e) Olber's Paradox
f) Rossiter-McLaughlin effect
g) Proper motion
h) Malmquist Bias
i) Blazar
j) Roche limit
k) Cosmological principle
l) Bias calibrations

PROBLEM 5

a) [3 pts] Define the specific intensity I_ν and flux density F_ν (in terms of I_ν). Draw schematic plots to show how you define the specific intensity.

b) [2 pts] Show that for an isotropic radiation field, $F_\nu = 0$.

c) [5 pts] Calculate the flux at a point P at an arbitrary distance r from a sphere of radius R with $I_\nu = B_\nu$ (see Figure below).



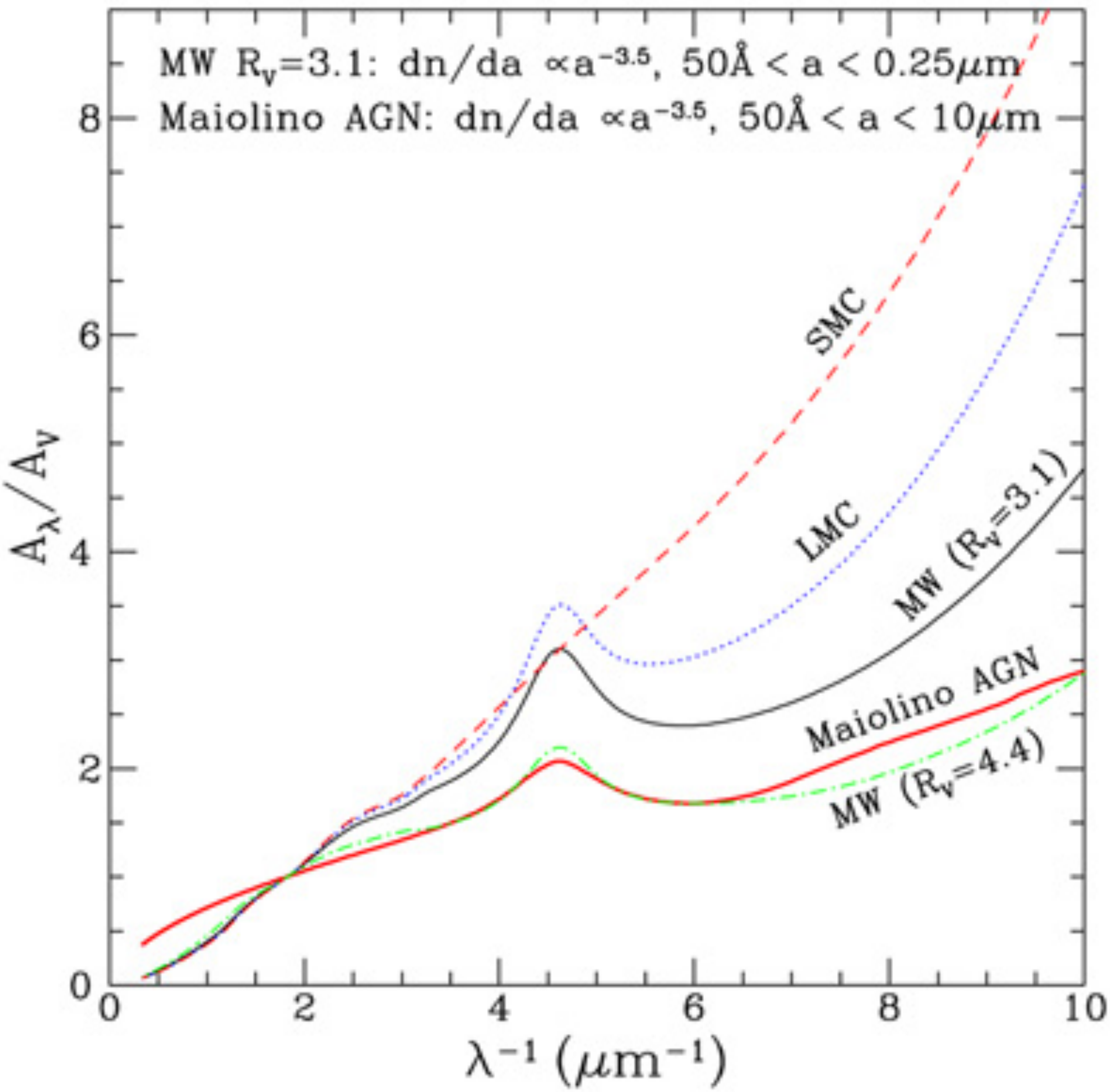
PROBLEM 6

a) [3 pts] Consider interstellar dust. Using the observed properties of dust, construct an argument for the content (i.e., elements and compounds) that they are made of.

b) [3 pts] In the vicinity of the sun, the energy density of starlight is $1.0 \times 10^{-12}\,\mathrm{erg\,cm^{-3}}$. Assuming a large (e.g., dust radius a $> 0.03\,\mu$m) dust grain, and that the dust emits and absorbs like a black body, determine the equilibrium temperature of the dust grain.

c) [1 pt] In fact, dust does not absorb and emit as a blackbody. Light is transmitted as an EM wave through the dust grain, and therefore, it can only efficiently absorb wavelengths smaller than the grain size. Will the dust therefore be cooler or warmer than the blackbody limit discussed in part b? Explain.

d) [3 pts] The ion He^{+} produces an emission line at 1640.4 Å, resulting from a transition from the 4th level to the third level, and another one at 4687 Å resulting from a transition from the third level to the second level. Atomic physics demands that the ratio of 1640.4 Å line to the 4687 Å line should be 6.6 under certain conditions. If the reddening in the AGN host galaxy is E(B-V)=0.1, what ratio of these two lines do you expect to measure? Use the reddening curve below and assume normal Milky Way dust with $R_V = 3.1$. For reference, B is centered around 4400Å and V is centered around 5500Å.



MW R_V=3.1: dn/da ∝$a^{-3.5}$, 50Å < a < 0.25μm
Maiolino AGN: dn/da ∝$a^{-3.5}$, 50Å < a < 10μm
SMC
LMC
MW (R_V=3.1)
Maiolino AGN
MW (R_V=4.4)
A_λ/A_V
λ^{-1} (μm^{-1})
0
2
4
6
8
10