ASTRONOMY QUALIFYING EXAM
January 2017

Possibly Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
a = 7.56 $\times 10^{-15}$ erg cm^{-3} K^{-4}
c = 3 $\times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
k = 1.38 $\times 10^{-16}$ erg K^{-1}
e = 4.8 $\times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
G = 6.67 $\times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s
1 amu = 1.66053886 $\times 10^{-24}$ g

PROBLEM 1

Use the Virial Theorem to:

a) (6 points) Derive the internal temperature of the Sun. How much hotter is this value compared to the Sun's effective surface temperature?

b) (4 points) Derive the Jeans Mass of a molecular cloud that is starting to collapse, thereby starting the star formation process.

PROBLEM 2

a) (4 points) The observed wavelength λ_0 is related to the emitted wavelength by

$$\lambda_0/\lambda = 1/R = 1 + z,$$

where R is the scale factor, and z is the redshift. The energy density in radiation from a black body is given by:

$$U(\nu, T)d\nu = 8\pi h\nu^3/c^2 \left(e^{h\nu/(kT)} - 1\right)^{-1} d\nu$$

Remembering that volumes increase like $V_0/V = 1/R^3$. Show that:

$$U(\nu_0, T)d\nu_0 = 8\pi h\nu_0^3/c^2 \left(e^{h\nu_0/(RkT)} - 1\right)^{-1} d\nu_0$$

b) (4 points) Given that

$$\int_0^\infty U(\nu, T)d\nu = aT^4$$

or equivalently

$$\int_0^\infty x^3/(e^x - 1)dx = \pi^4/15$$

show that the temperature of the Cosmic Background Radiation (CBR) must scale as 1/R.

c) (2 points) Compare the energy density in the CBR with that in diffuse starlight. Assume that the diffuse starlight has a brightness temperature of 10,000 K and a volume filling factor of 10^{-14}.

PROBLEM 3

The most easily observed white dwarf in the sky is in the constellation of Eridanus. Three stars make up the 40 Eridani system: 40 Eri A is a 4th magnitude star similar to the sun; 40 Eri B is a 10th magnitude white dwarf; and 40 Eri C is an 11th magnitude red M5 star. This problem deals only with the latter two stars, which are separated from 40 Eri A by 400 AU.

a) (4 points) The period of the 40 Eri B and C system is 247.9 years. The system's measured trigonometric parallax is 0.201 arcseconds, and the true angular extent of the semimajor axis of the reduced mass is 6.89 arc seconds. The ratio of the distances of 40 Eri B and C from the center of mass is $a_B/a_C = 0.37$. Find the masses of 40 Eri B and C in terms of the mass of the sun.

b) (2 points) The absolute bolometric magnitude of 40 Eri B is 9.6. Determine its luminosity in terms of the luminosity of the sun. Note that the absolute bolometric luminosity of the sun is $M_{bol} = 4.74$, while its luminosity is 3.839×10^{26} W.

c) (2 points) The effective temperature of 40 Eri B is 16,900 K. Calculate its radius and compare your answer with the radius of the Earth (6.378×10^6 m).

d) (2 points) Sirius B is another famous white dwarf star. It has a mass of $1.053\,M_\odot$. Do you expect it to be larger or smaller than 40 Eri B? Explain.

PROBLEM 4

You are planning to conduct high resolution optical spectroscopy toward Barnard's Star, whose current coordinates are α = 17:57:48.5 and δ=+04:41:36.2. It is 1.8 pc away from the Sun and it has a V-band magnitude of 9.51 (Vega system).

a) (1 point) What is constantly changing about Barnard's Star that needs to be considered when planning observations? What time of year is be to observed this star from the ground and why?

b) (1 point) What is the flux density of the star in V-band if the flux zero-point is 3.636×10^{-20} erg cm^{-2} s^{-1} Hz^{-1}?

c) (1 point) This observation will be source noise limited, what distribution describes the uncertainty of these measurements and what is the simplest equation for the uncertainty σ in this case (1 point).

d) (4 points) Derive an expression for the number of photons observed in a given Δt and calculate the number in a single resolution element for the ARCES spectrograph on the APO 3.5m at a wavelength of 5175 Angstroms. The ARCES spectrograph has a resolution of R$\sim$31,500 in the optical band. Assume that the V-band flux density is the flux density at 5175 Angstroms.

e) (1 point) What is the maximum exposure time to avoid detector non-linearity? The detector goes non-linear at 35,000 ADU and the gain of the detector is 3.8 e^{-} ADU^{-1}.

f) (2 points) Demonstrate mathematically that multiple short exposures are equivalent to a single long exposure. Why is a single long exposure a bad idea in the first place and why do we typically take multiple exposures during observations?

PROBLEM 5

Consider a satellite of mass m and radius s that is in a circular orbit about a planet with mass M and radius R. Assume the planet and satellite are separated by a distance r.

a) (3 points) Tidal forces arise because the gravitational force exerted by one body on another is not constant across it. For instance, something on the near edge of the satellite will feel a stronger gravitational pull toward the planet than the center of the satellite will. Thus, the tidal force is differential. Derive the tidal force (relative to the satellite's center) that a small object of mass u will feel if it is sitting on the edge of the satellite nearest to the planet. In this derivation, assume that $r \gg s$.

b) (3 points) Find the distance, d, from the planet where the tidal force that the small object experiences is equal to the gravitational pull exerted by the satellite's gravity.

c) (2 points) Express this distance in terms of the densities of the planet (ρ_{M}) and the satellite (ρ_{m}).

d) (2 points) Mars' moon Phobos has a density of 2 g/cm^3. It currently orbits Mars at a distance of 9400 km but this distance decreases by 2 cm every year. Using your work in parts a–c, calculate the amount of time before Phobos will be destroyed by the tidal forces of Mars. (The density and radius of Mars are 4 g/cm^3 and 3400 km, respectively.)

PROBLEM 6

Briefly define and discuss the relevance of the following terms to modern astronomy. 1 point per question

1. Cepheid variable star
2. Initial mass function
3. tunneling in the context of the PPI chain reaction
4. age-metallicity relation
5. damped Lyα system (DLA)
6. s-process
7. G dwarf problem
8. Tully-Fisher relation
9. Galactic thin disk
10. isophotal radius