Application of ARIMA and GARCH Models in Forecasting the Natural Gas Prices

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Abstract

Consumption of natural gas as the cleanest fossil fuel in the world is increasing dramatically. The natural gas market is very volatile, which results in a high level of risk for both consumers and producers. A more precise forecast tool enables us to hedge the financial risk. In this study, we attempt to predict natural gas spot price in the long term in the U.S. market using ARIMA/GARCH time series model. The results indicate, with a 95% confidence level, that the price will oscillate between 1.5 and 3.2 $/MMBtu through 2016.

In the last century, natural gas has been vented as a dangerous byproduct of the oil wells. Dissolved natural gas, which causes several problems in oil transportation, should be separated in order to reduce the transportation cost. For several decades, natural gas has imposed an extra cost to the oil industry and has been released to the environment. Natural gas is mainly composed of methane as well as some amounts of other heavier hydrocarbons, including ethane, propane, and butane. Some contaminants, such as sour gases (e.g. CO_2 and H_2S) and water contained in the natural gas should be removed from the stream before injecting it into a pipeline. By increasing the energy demand and its price, natural gas substituted other fuels, such as oil and coal. Since storage and transfer of natural gas in gaseous phase was not feasible for a long time, it used to be consumed locally near to the production fields. By development of pipelines and new transportation technologies, such as LNG and CNG, natural gas can be conveniently delivered to markets and remote areas across the world.

Since natural gas is the cleanest and the most abundant fossil fuel compared to other fossil resources in the world, by improving transportation technology and decreasing the handling costs, it is becoming the most popular source of energy globally. Focusing on environmental concerns and new regulations, power plant owners will opt to switch their fuel from coal and oil products to natural gas increasing the demand of natural gas even more in the future. Moreover, the rapid change in the economy of China needs a large amount of energy (U.S. EIA | IEO, 2013). Also, the global energy demand is increasing as well as the portion of demand of natural gas in the total energy consumption (BP, 2014).

However, new technologies, and explorations enabled us to produce more natural gas from tight and unconventional reservoirs to compensate the huge demand of energy in future.

In the early 1990s, natural gas financial markets started to operate when the regulations in the U.S. was adjusted in order to liberate the market, as well as national grid for natural gas transportation was expanded. Due to the extent of this evolution, reliable infrastructure for natural gas trading and availability has been implemented around the country.

In general the price of energy, natural gas in particular, is very volatile. For the investors who plan to build million-dollar power plants, the incoming cash flow involves a high degree of uncertainty because the fuel price flocculation brings instability in revenues. The price oscillation in short time denotes the risk of this commodity, which makes the investors worried about the fuel price as a major part of the operation costs.

Natural gas price prediction is important, as it will help to have a better picture of the market in the future and enables us to monitor the factors that might affect the price movement. There are several technics to model and predict the natural price trend. Since the natural gas price change is random, the stochastic process could explain the nature of this oscillation.

In addition, the seasonal nature of natural gas prices introduces further variables that can describe these fluctuations. Another method that could explain this random trend is the time series techniques. The price could be modeled based on various auto-regression methods. The artificial neural network came into the picture when computers became more popular. The use of fast computers enables us to run and train a neural network with high speed and accuracy.

Literature Review

Hotelling (1931) could be recognized as the first researcher who introduced a model to describe the behavior of exhaustible resources, such as natural gas. This model is known as Hotelling's rule; which states that the producers of a non-renewable commodity (e.g. Natural gas) tend to sell their commodities when the benefits from the sale are more than the benefits of keeping it. In the other words, the extraction cost and the present value of the commodity in one side, and the cost of the storage and future value of the commodity in other side, which are related by interest rate, will determine the proficiency of keeping or selling the commodity. It is assumed that the markets are efficient and the owners are motivated by the profit. This rule does not consider the emergence of new technologies and resources that may be explored and discovered in the future. Pindyck (1978) has optimized Hotelling's model for oil and gas case by taking the oil and gas reserve increment by exploration into account. An MIT Energy Lab report (MacAvoy & Pindyck, 1974) revealed the problem of natural gas shortages with econometrics models; which noted that the ceiling price was set by the Federal Power Commission and did not represent the price of the supply and demand equilibrium.

Bopp (1990) and Hsieh (1990) applied econometrics model, and Pilipovic (2007) implemented time series models to predict natural gas prices. Doris and Economides (1999) applied both econometrics and neural network models to forecast the natural gas price in the short term. Inikori, et al. (2001) investigated the effect of oil and natural gas prices on the oil and gas industry by establishing a linear regression model, which forecasted said prices.

Two forecasting models were developed by Nogales, et al. (2002) to predict the daily price of natural gas. They used the time series analysis approach to establish dynamic regression and transfer function for the Spain and California Markets, which resulted in the average errors of 5% and 3% respectively. Agbon and Araque (2003) applied chaos time series analysis and a nonlinear fuzzy neural network to predict the oil and gas prices. Ogwo, et al. (2007) developed an equitable pricing model to predict the natural gas price. Mishra (2012) modeled the natural gas price with time series as well as a nonparametric approach to forecast the price. Hu and Trafalis (2011) developed a new kernel for a neural network model to predict the natural gas price. Panella, et al. (2012) suggested a new approach to implement the neural network to model a nonlinear regression time series associated with energy commodity prices. Yi & Wang (2013) addressed the effect of oil prices on the international natural gas price using wavelet based Boltzmann cooperative neural network. Ekweanua, et al. (2014) found a robust correlation between the natural gas price and production, import, and export amount by means of a neural network model.

Several approaches and models have been proposed and applied by different researchers in order to predict natural gas prices in the long or short term, although each method and model does possess its own advantages and disadvantages. The multivariate models that take several variables into account are more accurate than the univariate models. However the external variables often themselves need to be predicted. For example, a predictor model for natural gas price, which is a function of oil price, struggles with the same uncertainty in oil price. We made the decision to model the natural gas monthly prices with a univariate nonlinear time series model, ARIMA/GARCH, to predict the price without any variable but with its lags.

Methodology

It is our objective in this paper to find a model to fit and predict the natural gas spot price at the Henry Hub. Natural gas price is a random variable that follows a stochastic process with a random trend. Initially, we define and bring attention to a few important words, which are pertinent in our modeling process.

Random Variables

A random variable is a measurable function of X from the probability space Ω into the set of real numbers $\mathbb{R}$ known as the state space. Three modifications are needed to make this definition more precise (Gallager, 2013):

1. The mapping $X(\omega)$ must have the property that $\{\omega \in \Omega : X(\omega) \leq x\}$ is an event for each $x \in \mathbb{R}$.
2. Every finite set of random variables $X_1, \dots, X_n$ has the property that for each $x_1 \in \mathbb{R}, \dots, x_n \in \mathbb{R}$, the set $\{\omega \in \Omega : X_1(\omega) \leq x_1, \dots, X_n(\omega) \leq x_n\}$ is an event.
3. X might be undefined or infinite for a subset of Ω that has 0 probability. In other words, the probability of events $\{X = \pm\infty\}$ is zero.

Stochastic Process

In probability theory, a stochastic process is a family of random variables X from the probability space Ω into $\mathbb{R}$ indexed by time set $t \in \mathbb{T}$, which could be denoted by $\{X(t, \omega): t \in \mathbb{T}, \omega \in \Omega\}$. We shall simplify the notation to $\{X(t): t \in \mathbb{T}\}$ or $\{X_t: t \in \mathbb{T}\}$ when the time variable t is continuous ($\mathbb{T} = \mathbb{R}$) or is a discrete variable ($\mathbb{T} = \mathbb{N}$) (Prabhu, 2007). In a stochastic or random process, there are some ambiguities: even if the initial condition (or starting point) is known, there are several directions in which the process may proceed.

Time Series

The time series is a stochastic process of random variable X indexed by time. X_t is a notation used for discrete parameter process and $X(t)$ is a notation for continuous parameter process. In this research, we are dealing with the discrete parameter process of time series.

Stationary Time Series

A time series is stationary if its statistical parameters such as mean and variance remain constant for all the time steps.

Autocorrelation and Partial Autocorrelation

Autocorrelation is a correlation of time series with itself. This function measures the correlation between the variable X_t and its lag X_{t-k} which is a real value between -1 and 1 where -1 implies a complete negative correlation and 1 is a complete positive correlation while 0 indicates no correlation between the variables. In a similar way, partial autocorrelation is a correlation between X_t and X_{t-k} when X_{t-k} comes into pictures and improves the correlation to X_t. On the other hands, partial autocorrelation of a variable in k order is the amount of correlation between the variable and its k^{th} lag that was not explained by the correlations at all lower orders lags.

Monte Carlo Simulation

Monte Carlo methods are a wide range of computational algorithms that rely on the repeated random sampling or output of a random-base function to obtain the best numerical results. In a stochastic process or time series simulation, the functions generate different results in different iterations. They are often used in physical, mathematical and statistical problems and are most useful when it is difficult or impossible to use the other mathematical methods.

ARMA Model

Autoregressive Moving Average process, ARMA (r, m), is a combination of Moving Average MA (m) and Autoregressive AR (r) processes. Suppose $\{X_t;\ t = \pm 1, \pm 2, \dots\}$ is a causal, stationary, and invertible process. Therefore, it satisfies the following equation:

$$X_t - \mu - \phi_1 (X_{t-1} - \mu) - \cdots - \phi_r (X_{t-r} - \mu)$$

$$= a_t - \theta_1 a_{t-1} - \cdots - \theta_m a_{t-m}$$

$$X_t - \mu - \phi_1 B^1 (X_t - \mu) - \cdots - \phi_r B^r (X_t - \mu) \qquad \textit{Eq. 1}$$

$$= a_t - \theta_1 B^1 a_t - \cdots - \theta_m B^m a_t$$

$$\boldsymbol{\phi}(B)(X_t - \mu) = \boldsymbol{\theta}(B) a_t$$

Another expression for ARMA (r, m) model is

$$X_t - \mu - \sum_{i=1}^{r} \phi_i X_{t-i} = a_t - \sum_{j=1}^{m} \theta_i a_{t-j} \qquad \textit{Eq. 2}$$

ARIMA Model

The Autoregressive Integrated Moving Average, ARIMA (r, d, m), process of orders r, d, and m is a process, X_t, whose differences $(1-B)^d X_t$ satisfy an ARMA (r, m) model that is a stationary model in which d is a non-negative integer. We use the following notation:

$$\boldsymbol{\phi}(B)(1-B)^d X_t = \boldsymbol{\theta}(B) a_t \qquad \textit{Eq. 3}$$

where all the roots of $\phi(z) = 0$ and $\theta(z) = 0$ are outside of the unit circle, and $\phi(z)$ and $\theta(z)$ have no common factors. Parameter d in the ARIMA model represents the dth difference of X_t to find a stationary time series. Assume that X_t is not stationary, then we can reproduce a new time series with differencing the original time series such as $X_t - X_{t-1}$for d = 1 and $(X_t - X_{t-1}) - (X_{t-1} - X_{t-2})$ for d = 2 and so on until a stationary time series is obtained.

ARIMA (0, 1, 0) is the famous random walk model as follows:

$$X_t = \mu + X_{t-1} \qquad \textit{Eq. 4}$$

ARCH Model

To address the conditional volatility behavior, (Engle, 1982) introduced the Autoregressive Conditional Heteroscedasticity (ARCH) model. ARCH (q) is defined based on an ARMA model in which the term a_t is a function of conditional variance. Let $\sigma^2_{t|t-1}$ be the conditional variance of X_t, and suppose

$$a_t = \sigma_{t|t-1}\varepsilon_t$$
$$\sigma^2_{t|t-1} = \alpha_0 + \alpha_1 a^2_{t-1} + \cdots + \alpha_q a^2_{t-q} \qquad \textit{Eq. 5}$$

In this equation, $0 \le \alpha_1 < 1$ and ε_t's are independent, identically distributed, zero mean and unit variance random variables that are independent of $a_{t-k}, k = 1,2, \dots$. We can formulate the ARCH (q) model as follows:

$$\begin{cases} a_t = \sigma_{t|t-1} . \varepsilon_t \\ \sigma_{t|t-1}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, a_{t-i}^2 \end{cases} \qquad \text{Eq. 6}$$

GARCH Model

GARCH (p, q) is a generalized ARCH (p, q) model introduced by (Bollerslev, 1986) and (Taylor, 2007) which includes the lagged terms of the conditional variances. This model is defined as

$$\sigma_{t|t-1}^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_q a_{t-q}^2 + \beta_1 \sigma_{t-1|t-2}^2 + \cdots + \beta_p \sigma_{t-p|t-p-1}^2 \qquad \text{Eq. 7}$$

The compact form of GARCH (p, q) is

$$\begin{cases} a_t = \sigma_{t|t-1} . \varepsilon_t \\ \sigma_{t|t-1}^2 = \alpha_0 + \sum_{j=1}^{p} \beta_j \, \sigma_{t-j|t-j-1}^2 + \sum_{i=1}^{q} \alpha_i \, a_{t-i}^2 \end{cases} \qquad \text{Eq. 8}$$

Results and Discussion

In this work, we considered the natural gas monthly spot price at Henry Hub as a time series and attempted to fit the best model of ARIMA/GARCH by using the Econometrics Toolbox™ of MATLAB® software. The data was collected from U.S. Energy Information Administration's website (EIA, 2016).

Estimation of Parameter d

To implement the time series models described above on the natural gas spot market, we need to make sure that the given time series is stationary. For non-stationary time series, we should apply either a linear or non-linear transformation to achieve a stationary trend. As the first step, we must calculate the autocorrelations (ACF) and partial autocorrelation functions (PACF) of the time series that indicate the stationary or non-stationary behavior of the variable. The second step is to use standardized tests such as KPSS (Kwiatkowski, et al., 1992), and augmented Dickey-Fuller tests (Perron, 1988) in order to confirm the stationary behavior of the variable statistically. In order to convert a non-stationary time series into a stationary version we must make a new time series in terms of differences of one lag. This process defines the differencing parameter “d” in ARIMA model.

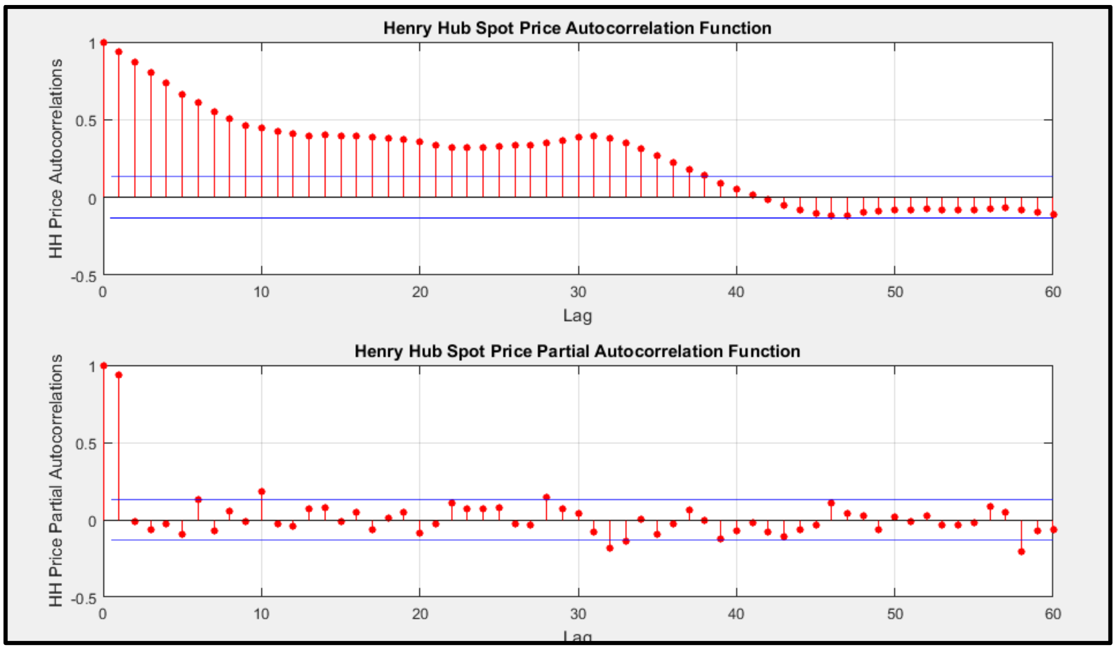

Figure 1. Autocorrelation and Partial Autocorrelations functions for Henry Hub Spot Price

As shown in Figure 4, the autocorrelation functions for original time series (i.e. Henry Hub spot price) do not converge to zero and have significant values for a large number of lags. They decay into the range very slowly and could not reach the domain even after 38 lags, but in the partial correlation graph after the second lag they become significantly smaller. It can be inferred from this behavior that this variable is not stationary. Therefore, we generated the new time series by differencing the natural gas price that is illustrated in Figure 5.

The KPSS and ADF tests results, summarized in Table 1, confirm our explanations of the AFC and PAFC graphs that the spot price trend is not a stationary time series but its first difference is. Therefore, we use the first difference (i.e. d =1) to establish an ARIMA-GARCH model.

Table 1. Result Summary for Stationary Tests

Variable	**Test**	**Stat.**	**Crit. Value**	**Likelihood Log**	**Significance Level**	**MSE**
Spot Price	KPSS	3.26	0.1460	-508.48	95%	5.110
	ADF	-1.25	-1.9421	-271.49	95%	0.643
First Difference of Price	KPSS	0.03	0.1460	-272.115	95%	0.612
	ADF	-15.42	-1.9421	-270.200	95%	0.643

Another method for estimation of the optimum number of differences is to find the minimum standard deviation of the produced time series, which is demonstrated in Figure 6. This method also leads us to d = 1.

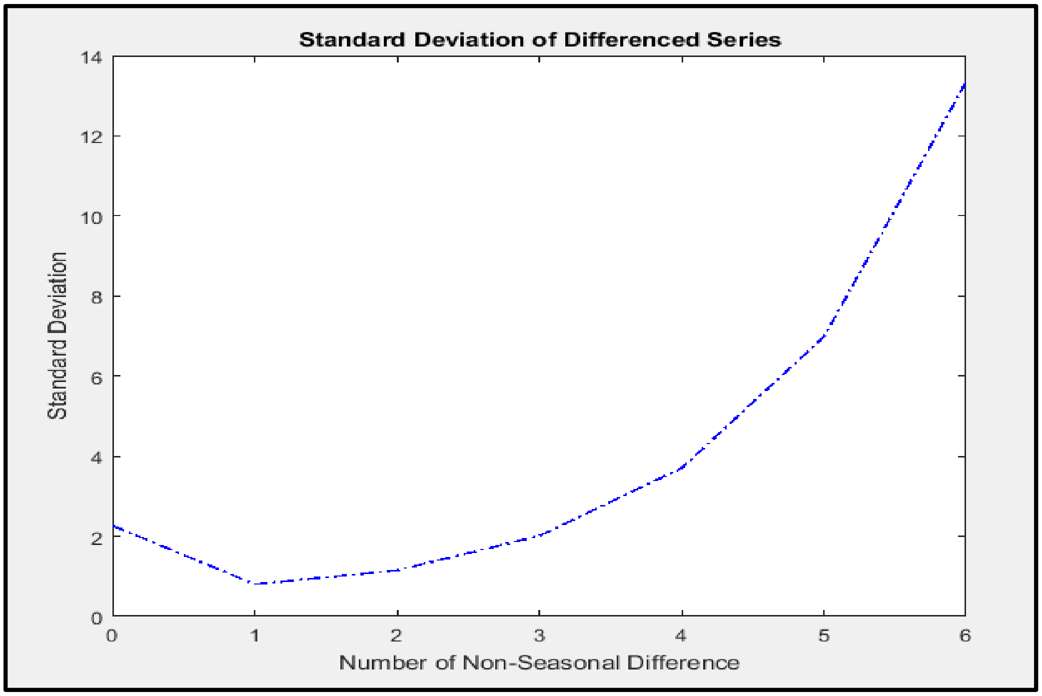

Figure 2. Standard Deviation of Differenced Time Series

Estimation of Parameters r and m

Figure 7, which illustrates the ACF and PACF for the model variable, indicates that the 5th and 9th lags of the time series have a close relationship with the variable. The negative values of these significant lags in PACF denote that we have a slightly over-differenced variable, which could be corrected by considering the lags as MA lags. Therefore, we may choose ARIMA (5,1,9) in which the coefficient for the lags are zero except the 5th and 9th lags. These relationships are not strong and the values are marginal. In order to determine the lags, several possibilities were tested by AIC and BIC; the most important results are listed in the Table 2 and Table 3 below.

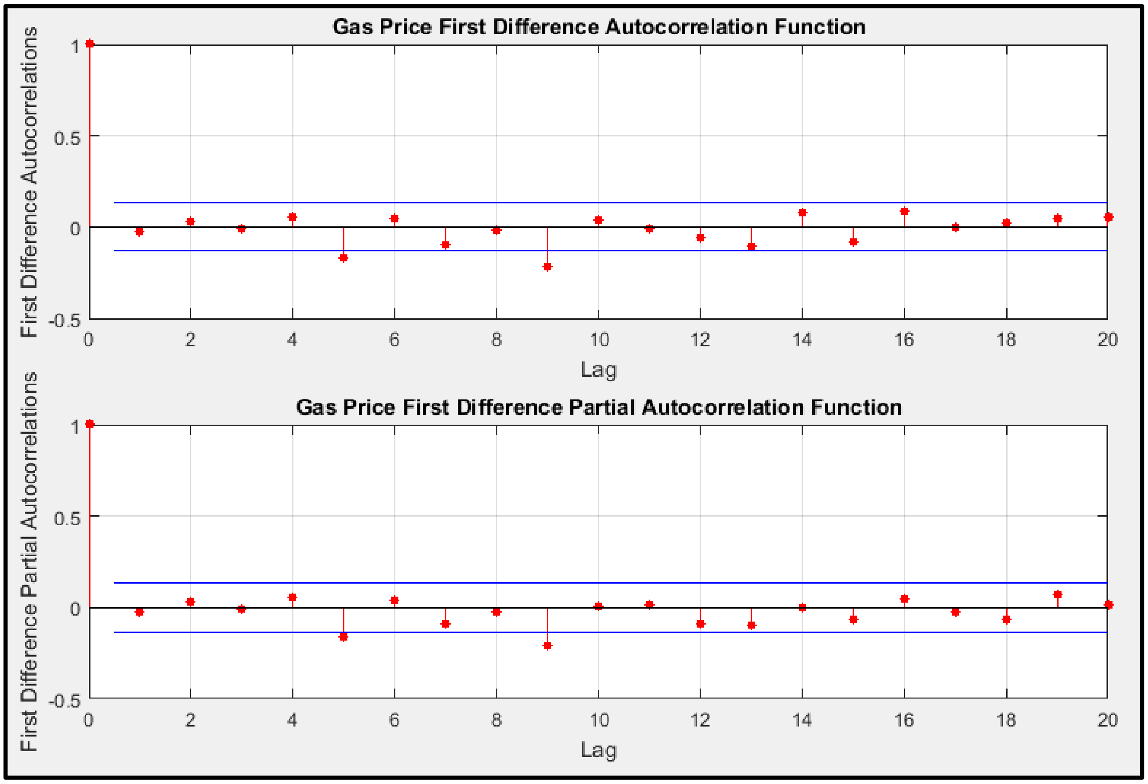

Figure 3. Autocorrelation and Partial Autocorrelations functions for First Difference of Spot Price

In order to find the optimum parameters of the ARIMA model, we defined various combinations of the lags and examined Akaike and Bayesian information criteria for those to choose the best model (Box, et al., 2015). According to the AIC results in Table 2, the ARIMA (9, 1, 9) model with 5th and 9th lags in both AR and MA section is the optimum model. The BIC results in Table 3 suggest choosing the ARIMA (5, 1, 9) model with 5th lag in AR and 9th lag in MA section. As the results of both methods for these two models are very close, we prefer to select the ARIMA (5,1,9) model that has two parameters less than the other model. A simpler model is always better if the accuracy is not significantly decreased.

Table 2. Akaike Information Criteria Results for Different Lags in ARIMA Model

		MA Lags				
		0	5	5, 9	9	1-9
AR Lags	0	549.92	545.76	536.65	539.56	547.53
	5	545.39	545.25	536.20	535.41	548.71
	5, 9	536.23	538.10	531.86	535.16	550.21
	9	540.16	537.04	537.46	542.08	548.56
	1-9	547.76	549.07	544.18	547.59	540.43

Table 3. Bayesian Information Criteria Results for Different Lags in ARIMA Model

		MA Lags				
		0	5	5, 9	9	1-9
AR Lags	0	556.77	556.04	550.36	549.85	585.26
	5	555.68	558.97	553.34	549.13	589.87
	5, 9	549.94	555.24	552.44	552.31	594.79
	9	550.45	550.76	554.61	555.80	589.71
	1-9	585.52	590.22	588.76	588.44	609.02

Estimation of ARCH/GARCH parameters

As shown in Figure 8, the residuals and squared residuals change along the time axis. In order to examine the heteroscedasticity effect, Ljung-Box Q-test was applied. The stat was 84.95 that compare to the critical value (31.41) confirms the illustration on the figure. These changes in statistical parameters led us to define a GARCH model for fitting the variances.

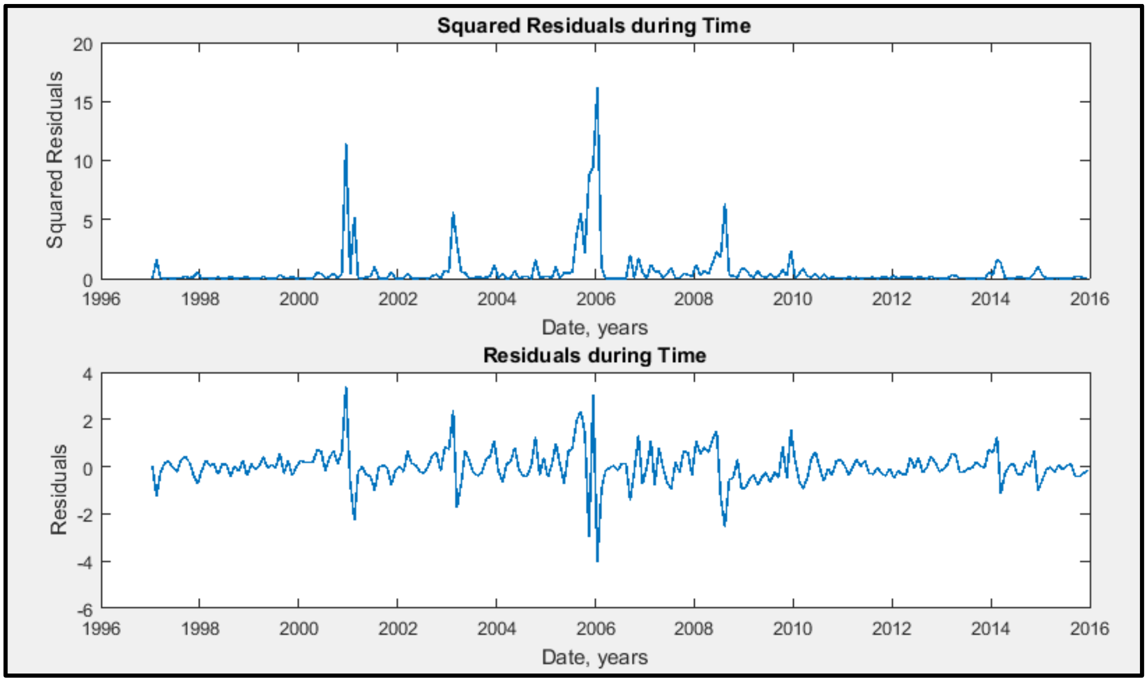

Figure 4. Squared Residuals during the Time

Figure 9 indicates that we have a significant arch effect in 2 lags of the modeling variable. The number of lags is the summation of ARCH and GARCH lags (p+q) together. We define GARCH (1, 1) for variance modeling and now our model is completed as ARIMA (5, 1, 9) / GARCH (1, 1). There exists a number of statistical tests, such as the Engle's test, which indicates the conditional heteroscedasticity of the time series (Engle, 1982).The stat of the Engle test for the first and second lags are 40.52 and 51.22 respectively, which compares to the critical values of 3.84 and 5.99, infers that the ARCH/GARCH effect is available.

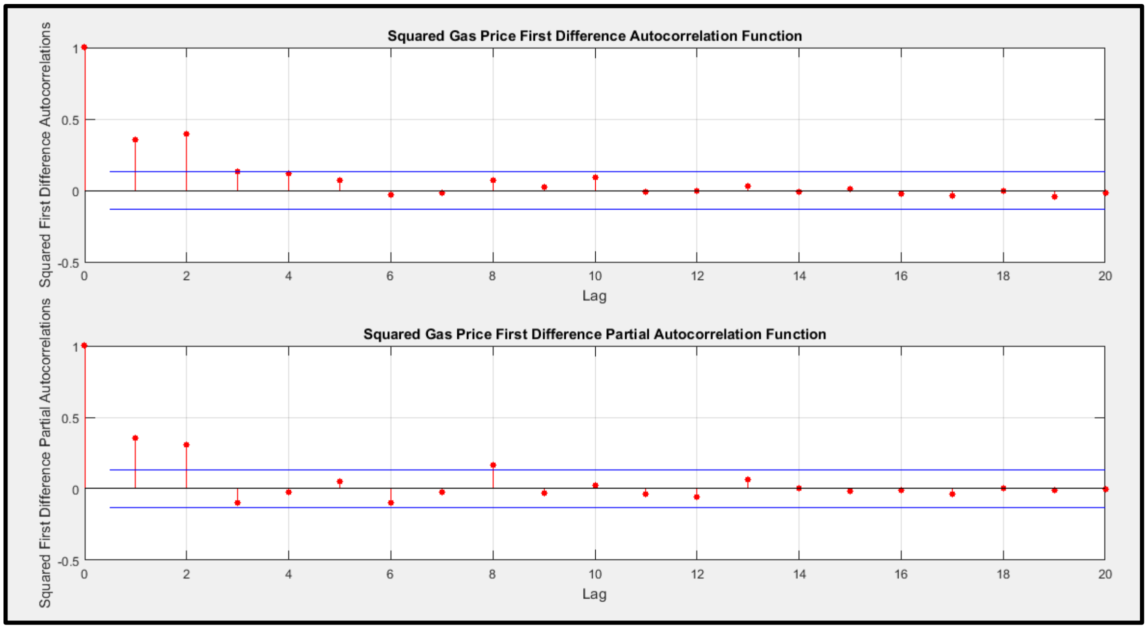

Figure 5. ACF and PACF of Squared First Difference of HH Spot Price

Table 4 shows the model parameters' values, standard errors, and t statistics associated with the parameters. Plugging the parameters in the model results in:

$$\begin{cases} X_t = X_{t-1} - 0.0279 - 0.0567\,(X_{t-5} - X_{t-6}) + a_t + 0.2205\, a_{t-9} \\ a_t = \sigma_{t|t-1} . \varepsilon_t \\ \sigma^2_{t|t-1} = 0.0499 + 0.5288\, \sigma^2_{t-1|t-2} + 0.4712\, a^2_{t-1} \end{cases} \quad \text{Eq. 9}$$

Table 4. Model's Parameters

Parameter	**Value**	**Std. Error**	**t Statistic**
ARIMA Const.	-0.0279	0.0230	-1.2162
AR(5)	-0.0567	0.0447	-1.2677
MA(9)	-0.2205	0.0344	6.4023

GARCH Const.	0.0499	0.0204	2.4433
ARCH(1)	0.4712	0.0725	6.5023
GARCH(1)	0.5288	0.0653	8.0958

To validate and check the model, we ran the model starting from 1999 until the last date of our available data.

We first established the model for the early data, then later rebuilt it for the emerging date over time, as well as forecasted the price in the subsequent step. The results are illustrated in Figure 10. In Figure 10 (a), the dotted red line represents the forecasted prices, while the solid blue line represents the actual historical data. In Figure 10 (b) the residual values are illustrated. The residual values indicate that the model is outperforming when new data enters the model. The residual values converge to zero over time, however the model has poor performance at the spikes.

Finally, with a confidence level of 95%, we ran the model to predict the price for 12 months in the future. The results are demonstrated in Figure 11. The prediction shows a descending trend until it reaches the lowest point in a price of 2 $/MMBtu on May 2016, and then starts to move upward until it hits the price of 2.3 $/MMBtu on September 2016. It also shows that the natural gas price will fluctuate between 1.5 and 3.2 $/MMBtu in 95% level of confidence.

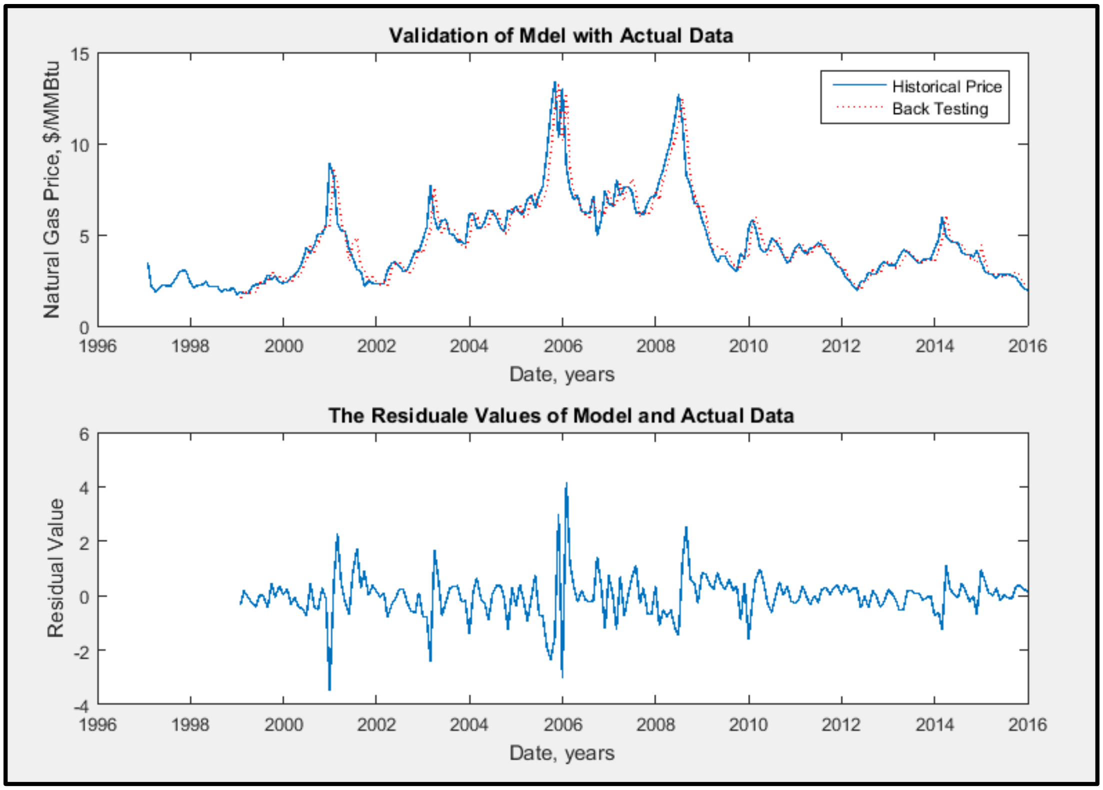

Figure 6. (a) Model Back Test Results, (b) Residual Value for Model

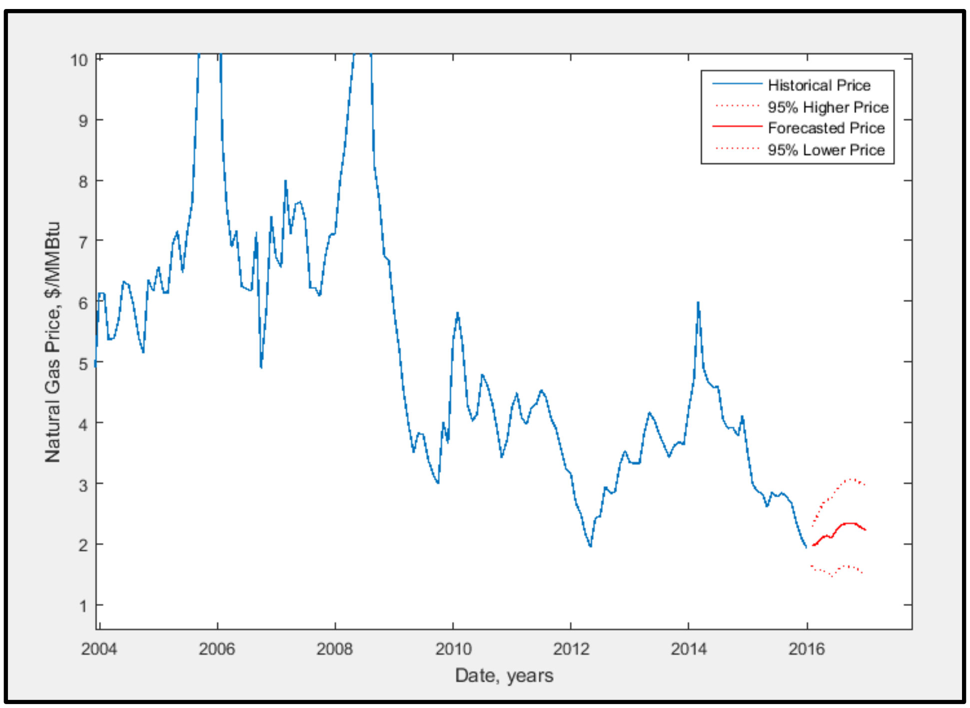

Figure 7. Model Forecasting for 12 Months in Future

Conclusions

The applied model here, the combination of ARIMA (5, 1, 9) and GARCH (1, 1) models, is a univariate time series model; it explains the history of a time series using its lag's variables. The univariate models do not need extra variables to predict the future prices and this helps us to avoid the uncertainty involved in those new variables.

The ARIMA (5, 1, 9) / GARCH (1, 1) model, shows that the natural gas price is likely to increase slightly in the future, which is not significant and cannot exceed 3.2 $/MMBtu even in the optimistic case. The forecast graph indicates that in 2016, the natural gas price tends to remain above 1.5 $/MMBtu. In other words, by 95% level of confidence, the natural gas price will fluctuate between 1.5 and 3.2 $/MMBtu. The probable value for the price is within 2 – 2.3 $/MMBtu, with the highest value in May 2016 and the lowest value in September.

The back test's resulting residuals demonstrate that by introducing new data in the model, which affects the variance model, the prediction error will decrease. Based on the measured errors, we can conclude that the model has accurate predictions, with the exception of the spikes.

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