DESIGN AND PLANNING UNDER UNCERTAINTY

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CHE 4273

Two-Stage
Stochastic Optimization Models

Philosophy

- Maximize the *Expected Value* of the objective over all possible realizations of uncertain parameters.
- Typically, the objective is *Profit* or *Net Present Value*.
- Sometimes the minimization of *Cost* is considered as objective.

Uncertainty

- Typically, the uncertain parameters are: *market demands, availabilities, prices, process yields, rate of interest, inflation, etc.*
- In Two-Stage Programming, uncertainty is modeled through a finite number of independent *Scenarios*.
- Scenarios are typically formed by *random samples* taken from the probability distributions of the uncertain parameters.

Characteristics of Two-Stage Stochastic Optimization Models

First-Stage Decisions

- Taken before the uncertainty is revealed. They usually correspond to structural decisions (not operational).
- Also called "Here and Now" decisions.
- Represented by "Design" Variables.
- Examples:
  - To build a plant or not. How much capacity should be added, etc.
  - To place an order now.
  - To sign contracts or buy options.
  - To pick a reactor volume, to pick a certain number of trays and size the condenser and the reboiler of a column, etc

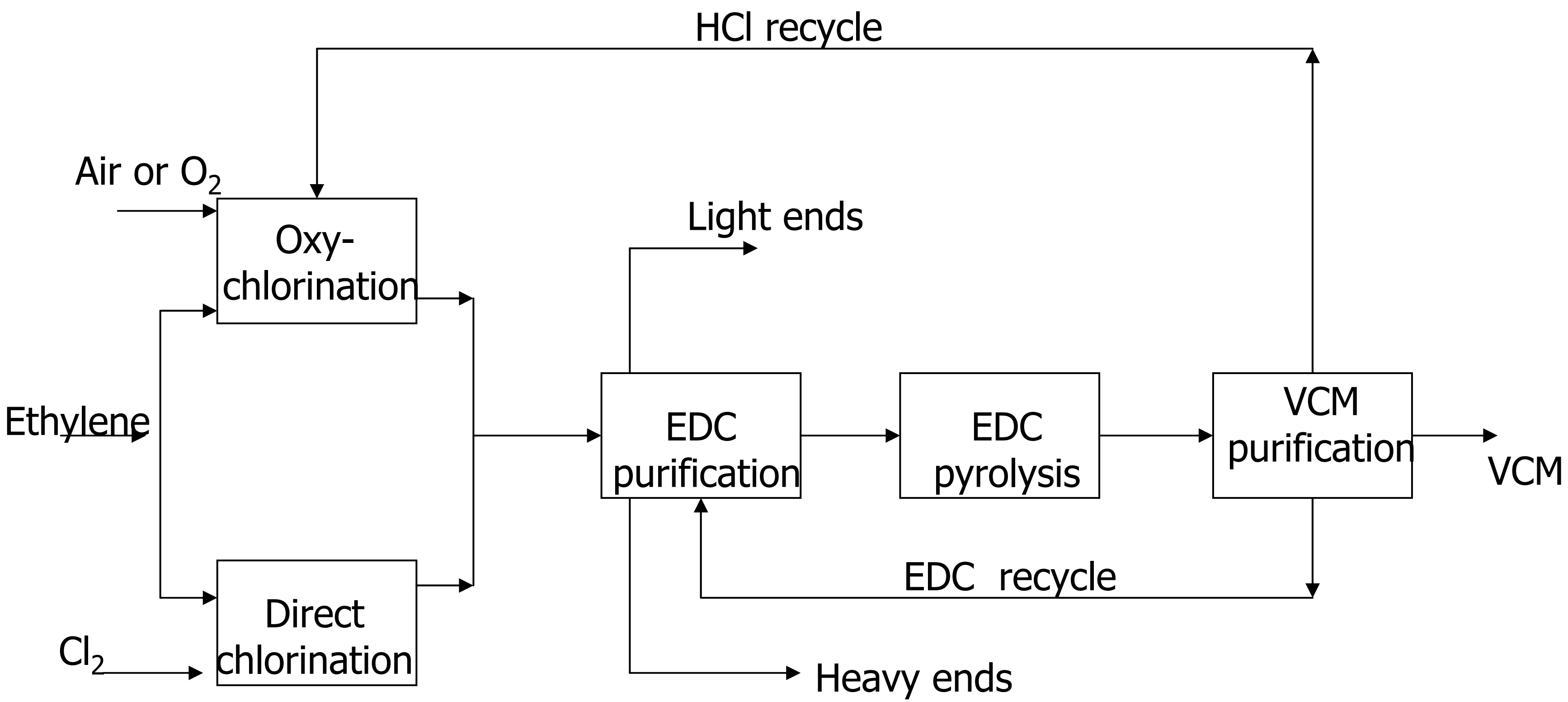
Characteristics of Two-Stage Stochastic Optimization Models

Second-Stage Decisions

- Taken in order to adapt the plan or design to the uncertain parameters realization.
- Also called "Recourse" decisions.
- Represented by "Control" Variables.
- Example: the operating level; the production slate of a plant.

- Sometimes first stage decisions can be treated as second stage decisions. In such case the problem is called a multiple stage problem.

Example: Vinyl Chloride Plant



Example: Vinyl Chloride Plant

Consider the following forecasts:

Forecasted prices of raw materials product

Year	Ethylene	Chlorine	Oxygen	VCM
	$/ton	$/ton	$/ft3	$/ton
2004	492.55	212.21	0.00144	499.19
2005	499.39	214.14	0.00144	506.19
2006	506.22	216.07	0.00143	513.18
2007	513.06	218.00	0.00142	520.18
2008	519.90	219.93	0.00141	527.18
2009	526.73	221.86	0.00140	529.17
2010	533.57	223.79	0.00139	535.17
2011	540.41	225.72	0.00138	543.17
Std. Dev	24.17	10.56	0.00010	26.15

Forecasted excess demand over current capacity

Year	VCM
	lb-mol/hr
2004	3602
2005	5521
2006	7355
2007	9551
2008	11888
2009	14322
2010	16535
2011	18972

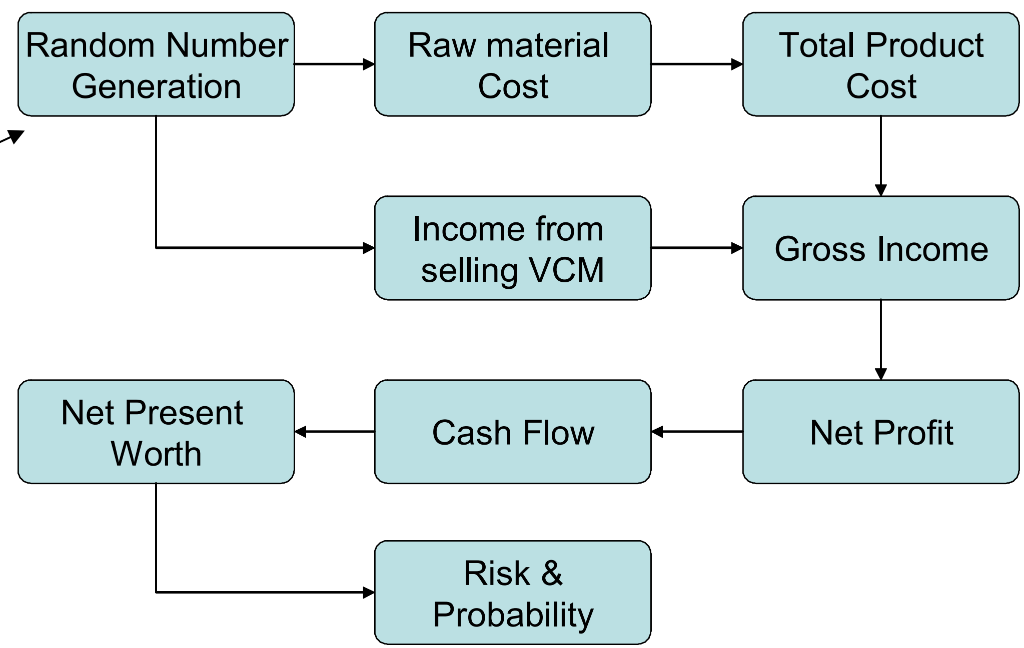
Consider building (in 2004) for three capacities to satisfy excess demand at 2004, 2006 and 2011. Plants will operate under capacity until 2006 or 2011 in the last two cases. These are 3 different first stage decisions.

Example: Vinyl Chloride Plant

The different investment costs are:

Plant Capacity	4090 MMlb/yr	6440 MMlb/yr	10500 MMlb/yr
TCI	$47,110,219	$68,886,317	$77,154,892

Consider the following calculation procedure

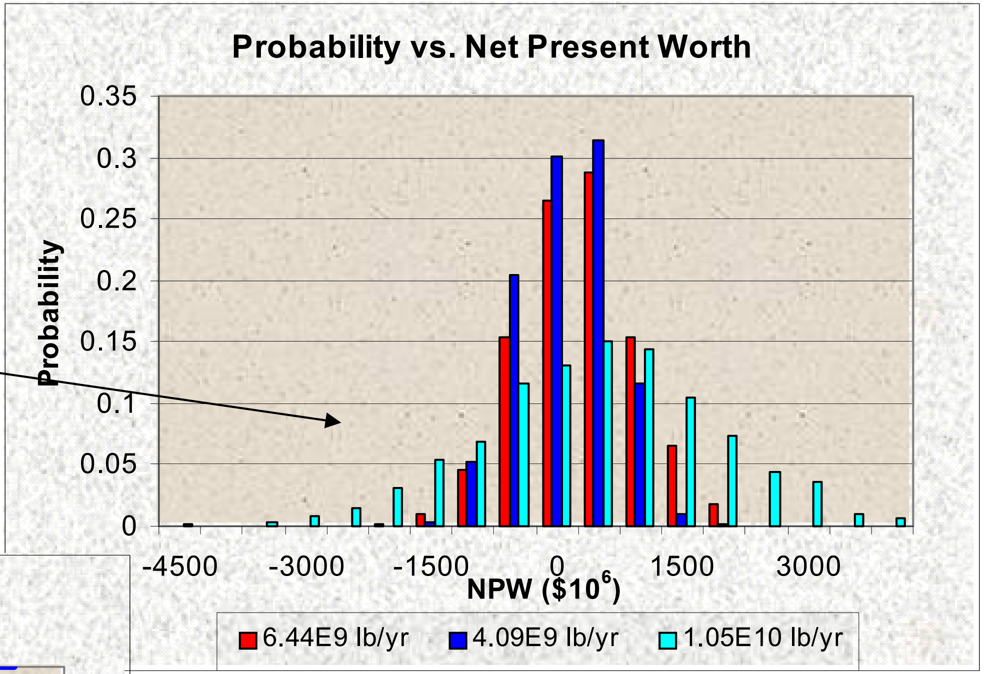


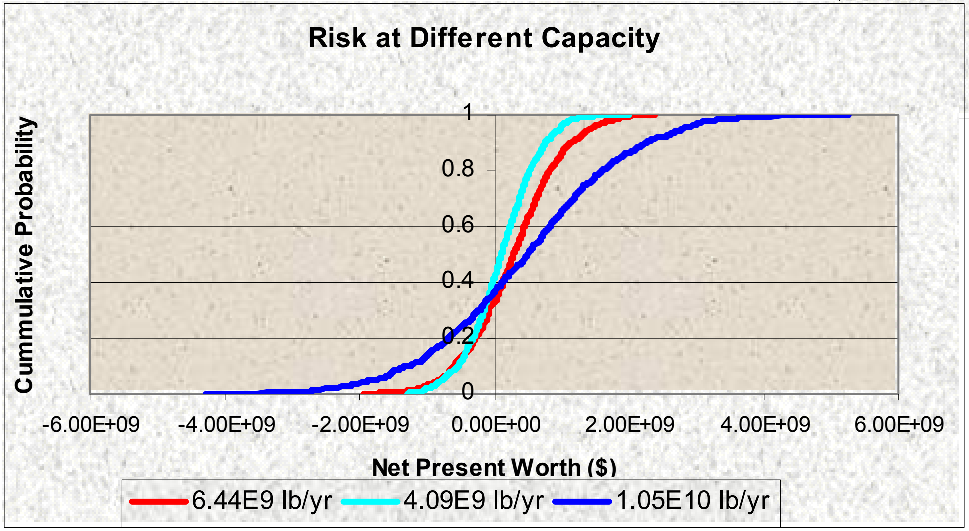
Random numbers are obtained for each year for raw materials and product prices using sampling from a normal distribution. This can be done in Excel

Example: Vinyl Chloride Plant

Histograms and Risk Curves are

Notice the asymmetry in the distributions.





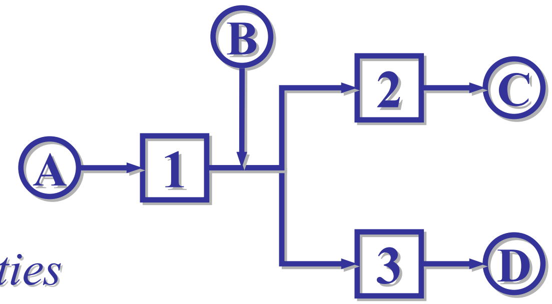
The risk curves show a 36% chance of losing money for the 10.5 billion lbs/year capacity, 31.7% for the 6.44 billion lbs/yr capacity and 41% chance for the 4.09 billion lbs/year capacity.

Expected Profits are: 24%, 25% and 20%.

Capacity Planning

GIVEN:

- **Process Network**
  - *Set of Processes*
  - *Set of Chemicals*



- **Forecasted Data**
  - *Demands & Availabilities*
  - *Costs & Prices*
  - *Capital Budget*

DETERMINE:

- **Network Expansions**
  - *Timing*
  - *Sizing*
  - *Location*
- **Production Levels**

OBJECTIVES:

- **Maximize Net Present Value**

Capacity Investment Planning

Design Variables: to be decided before the uncertainty reveals

$$x = \{ Y_{it}, E_{it}, Q_{it} \}$$

Y: Decision of building process *i* in period *t*
E: Capacity expansion of process *i* in period *t*
Q: Total capacity of process *i* in period *t*

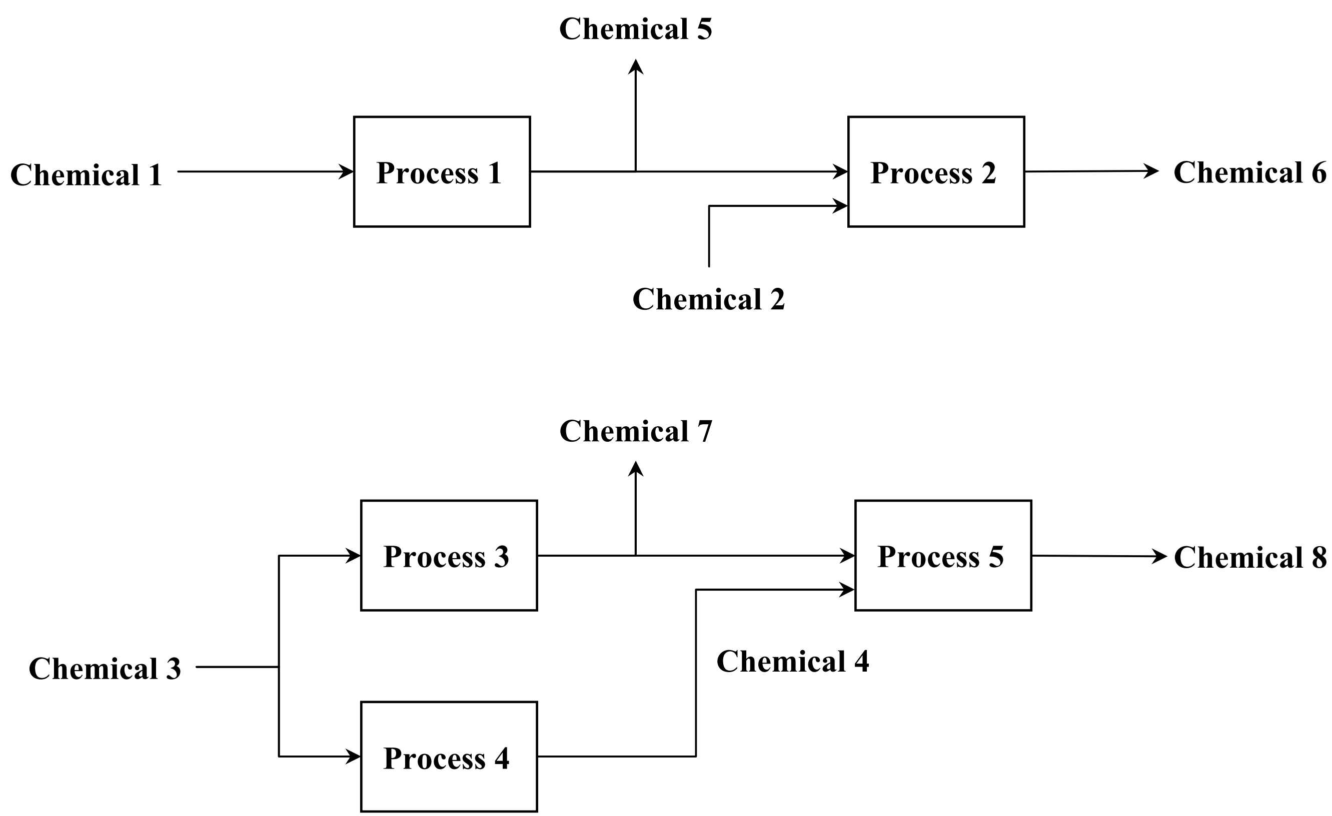
Control Variables: selected after uncertain parameters become known. Assume them known for the time being!!!!

$$y_s = \{ S_{jlt}, P_{jlt}, W_{it} \}$$

S: Sales of product *j* in market *l* at time *t* and scenario *s*
P: Purchase of raw mat. *j* in market *l* at time *t* and scenario *s*
W: Operating level of of process *i* in period *t* and scenario *s*

Example

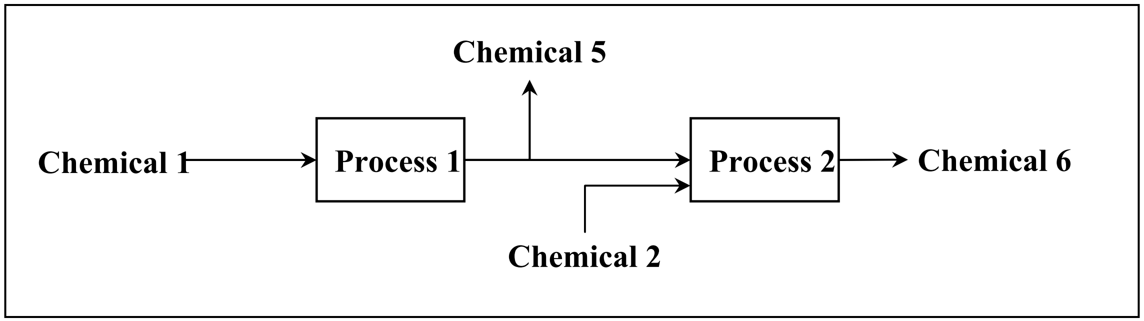
- Project Staged in 3 Time Periods of 2, 2.5, 3.5 years



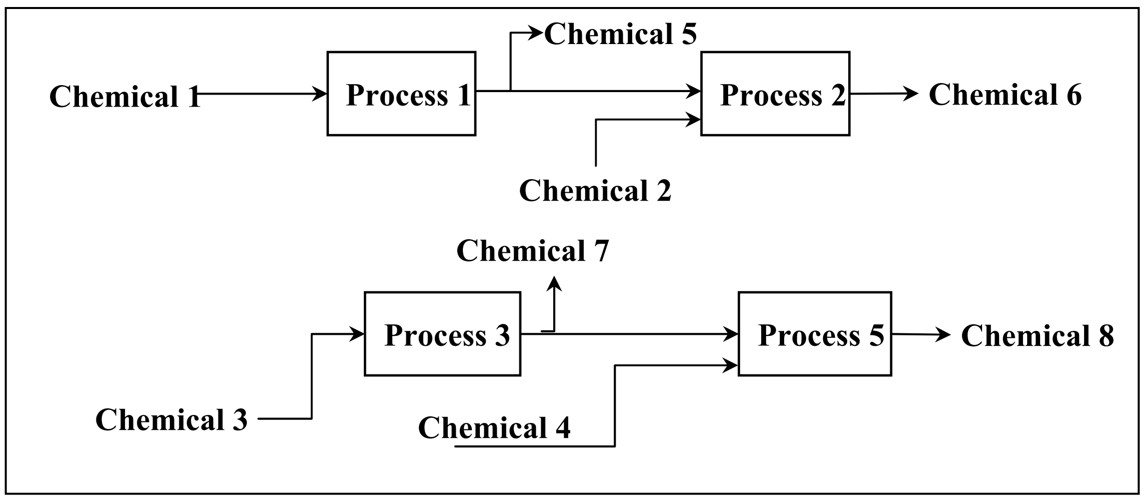
Example

- One feasible (not necessarily optimal) solution could be

Period 1
2 years



Period 2
2.5 years



Period 3
3.5 years

Same flowsheet different production rates

MODEL

SETS

I : Processes i,=1,…,NP
J : Raw materials and Products, j=1,…,NC
T: Time periods. T=1,…,NT
L: Markets, l=1,..NM

VARIABLES

Y_{it}: *An expansion of process I in period t takes place (Yit=1), does not take place (Yit=0)*
E_{it}: *Expansion of capacity of process i in period t.*
Q_{it}: *Capacity of process i in period t.*
W_{it}: *Utilized capacity of process i in period t.*
P_{jlt} : *Amount of raw material/intermediate product j consumed from market l in period t*
S_{jlt} : *Amount of intermediate product/product j sold in market l in period t*

PARAMETERS

η_{ij} : *Amount of raw material/intermediate product j used by process i*
μ_{ij} : *Amount of product/intermediate product j consumed by process i*
γ_{jlt} : *Sale price of product/intermediate product j in market l in period t*
Γ_{jlt} : *Cost of product/intermediate product j in market l in period t*
δ_{it} : *Operating cost of process i in period t*
α_{it} : *Variable cost of expansion for process i in period t*
β_{it} : *Fixed cost of expansion for process i in period t*
L_t : *Discount factor for period t*
E_{it}^{L}, E_{it}^{U} :*Lower and upper bounds on a process expansion in period t*
a_{jlt}^{L}, a_{jlt}^{U} : *Lower and upper bounds on availability of raw material j in market l in period t*
d_{jlt}^{L}, d_{jlt}^{U} : *Lower and upper bounds on demand of product j in market l in period t*
CI_t : *Maximum capital available in period t*
$NEXP_t$: *maximum number of expansions in period t*

MATHEMATICAL PROGRAMMING MODEL

OBJECTIVE FUNCTION

$$Max\ \ NPV = \sum_{t=1}^{NT} L_t \left(\sum_{l=1}^{NM} \sum_{j=1}^{NC} \left(\gamma_{jlt} S_{jlt} - \Gamma_{jlt} P_{jlt} \right) - \sum_{i=1}^{NP} \delta_{it} W_{it} - \sum_{i=1}^{NP} \sum_{t=1}^{NT} \left(\alpha_{it} E_{it} + \beta_{it} Y_{it} \right) \right)$$

DISCOUNTED REVENUES *INVESTMENT*

Y_{it}: An expansion of process I in period t takes place (Yit=1), does not take place (Yit=0)
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L_t : Discount factor for period t

MODEL

LIMITS ON EXPANSION $Y_{it}E_{it}^{L} \le E_{it} \le Y_{it}E_{it}^{L}$ $\quad i=1,\ldots,NP \quad t=1,\ldots,NT$

TOTAL CAPACITY IN EACH PERIOD $Q_{it} = Q_{i(t-1)} + E_{it}$ $\quad i=1,\ldots,NP \quad t=1,\ldots,NT$

LIMIT ON THE NUMBER OF EXPANSIONS $\sum_{t=1}^{NT} Y_{it} \le NEXP_i$ $\quad i=1,\ldots,NP$

LIMIT ON THE CAPITAL INVESTMENT $\sum_{i=1}^{NP} (\alpha_{it}E_{it} + \beta_{it}Y_{it}) \le CI_t$ $\quad t=1,\ldots,NT$

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α_{it} : Variable cost of expansion for process i in period t
β_{it} : Fixed cost of expansion for process i in period t
E_{it}^{L}, E_{it}^{U} : Lower and upper bounds on a process expansion in period t

MODEL

UTILIZED CAPACITY IS LOWER THAN TOTAL CAPACITY

$$W_{it} \leq Q_{it} \qquad i=1,\ldots,NP \quad t=1,\ldots,NT$$

MATERIAL BALANCE

$$\sum_{l=1}^{NM} P_{jlt} + \sum_{i=1}^{NP} \eta_{ij} W_{it} = \sum_{l=1}^{NM} S_{jlt} + \sum_{i=1}^{NP} \mu_{ij} W_{it} \qquad i=1,\ldots,NP \quad t=1,\ldots,NT$$

BOUNDS

$$a_{jlt}^{L} \leq P_{jlt} \leq a_{jlt}^{U} \qquad d_{jlt}^{L} \leq S_{jlt} \leq d_{jlt}^{U} \qquad j=1,\ldots,NC\ , t=1,\ldots,NT\ , l=1,\ldots,NM$$

NONNEGATIVITY

$$E_{it}, Q_{it}, W_{it}, P_{jlt}, S_{jlt} \geq 0 \qquad \forall i, j, l, t$$

INTEGER VARIABLES

$$Y_{it} \in \{0,1\} \qquad i=1,\ldots,NP \quad t=1,\ldots,NT$$

Y_{it}: An expansion of process I in period t takes place (Yit=1), does not take place (Yit=0)
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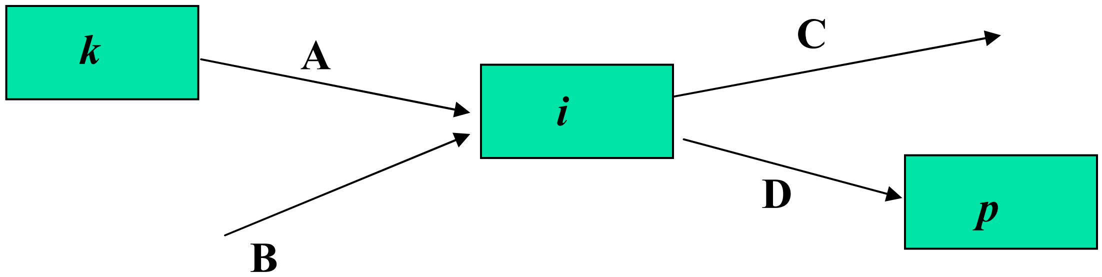
a_{jlt}^{L}, a_{jlt}^{U} : Lower and upper bounds on availability of raw material j in market l in period t

d_{jlt}^{L}, d_{jlt}^{U} : Lower and upper bounds on demand of product j in market l in period t

MODEL

MATERIAL BALANCE $\sum_{l=1}^{NM} P_{jlt} + \sum_{i=1}^{NP} \eta_{ij} W_{it} \leq \sum_{l=1}^{NM} S_{jlt} + \sum_{i=1}^{NP} \mu_{ij} W_{it}$ $\quad i=1,\ldots,NP \quad t=1,\ldots,NT$



$$\sum_{l=1}^{NM} P_{Blt} + \eta_{kA} W_{kt} = \sum_{l=1}^{NM} S_{Clt} + \mu_{iD} W_{it}$$

η_{kA}, μ_{iD}

"Stoichiometric" Coefficients

Reference Component is C

Two-Stage Stochastic Formulation

Let us leave it linear because as is it is complex enough.!!!

LINEAR MODEL SP

$$Max \sum_s p_s q_s^T y_s - c^T x$$

Recourse Function — **First-Stage Cost**

$s.t.$

$$Ax = b$$ **First-Stage Constraints**

$$T_s x + W y_s = h_s$$ **Second-Stage Constraints**

$$x \geq 0 \qquad x \in X$$

$$y_s \geq 0$$

Technology matrix

First stage variables

Second Stage Variables

Recourse matrix (Fixed Recourse)

Sometimes not fixed (Interest rates in Portfolio Optimization)

Complete recourse**:** the recourse cost (or profit) for every possible uncertainty realization remains finite, independently of the first-stage decisions (x).

Relatively complete recourse**:** the recourse cost (or profit) is feasible for the set of feasible first-stage decisions. This condition means that for every feasible first-stage decision, there is a way of adapting the plan to the realization of uncertain parameters.

We also have found that one can sacrifice efficiency for certain scenarios to improve risk management. We do not know how to call this yet.

Capacity Planning Under Uncertainty

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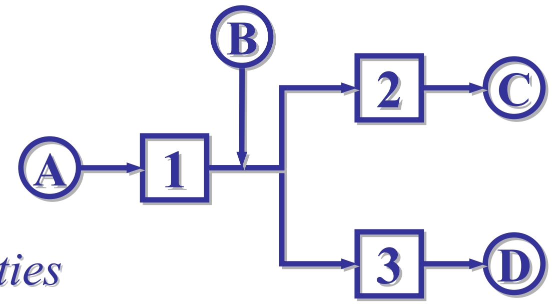
  - *Set of Processes*
  - *Set of Chemicals*



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DETERMINE:

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  - *Timing*
  - *Sizing*
  - *Location*
- **Production Levels**

OBJECTIVES:

- **Maximize Expected Net Present Value**
- **Minimize Financial Risk**

Process Planning Under Uncertainty

Design Variables: to be decided before the uncertainty reveals

$$x = \{ Y_{it}, E_{it}, Q_{it} \}$$

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$$y_s = \{ S_{jlts}, P_{jlts}, W_{its} \}$$

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BOUNDS

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d^L_{jlts}, d^U_{jlts} : *Lower and upper bounds on demand of product j in market l in period t, scenario s*

MODEL

OBJECTIVE FUNCTION

$$Max\ \ NPV = \underbrace{\sum_{s} p_s \left\{ \sum_{t=1}^{NT} L_t \left(\sum_{l=1}^{NM} \sum_{j=1}^{NC} \left(\gamma_{jlts} S_{jlts} - \Gamma_{jlts} P_{jlts} \right) - \sum_{i=1}^{NP} \delta_{its} W_{its} \right) \right\}}_{\textit{DISCOUNTED REVENUES}} - \underbrace{\sum_{i=1}^{NP} \sum_{t=1}^{NT} \left(\alpha_{it} E_{it} + \beta_{it} Y_{it} \right)}_{\textit{INVESTMENT}}$$

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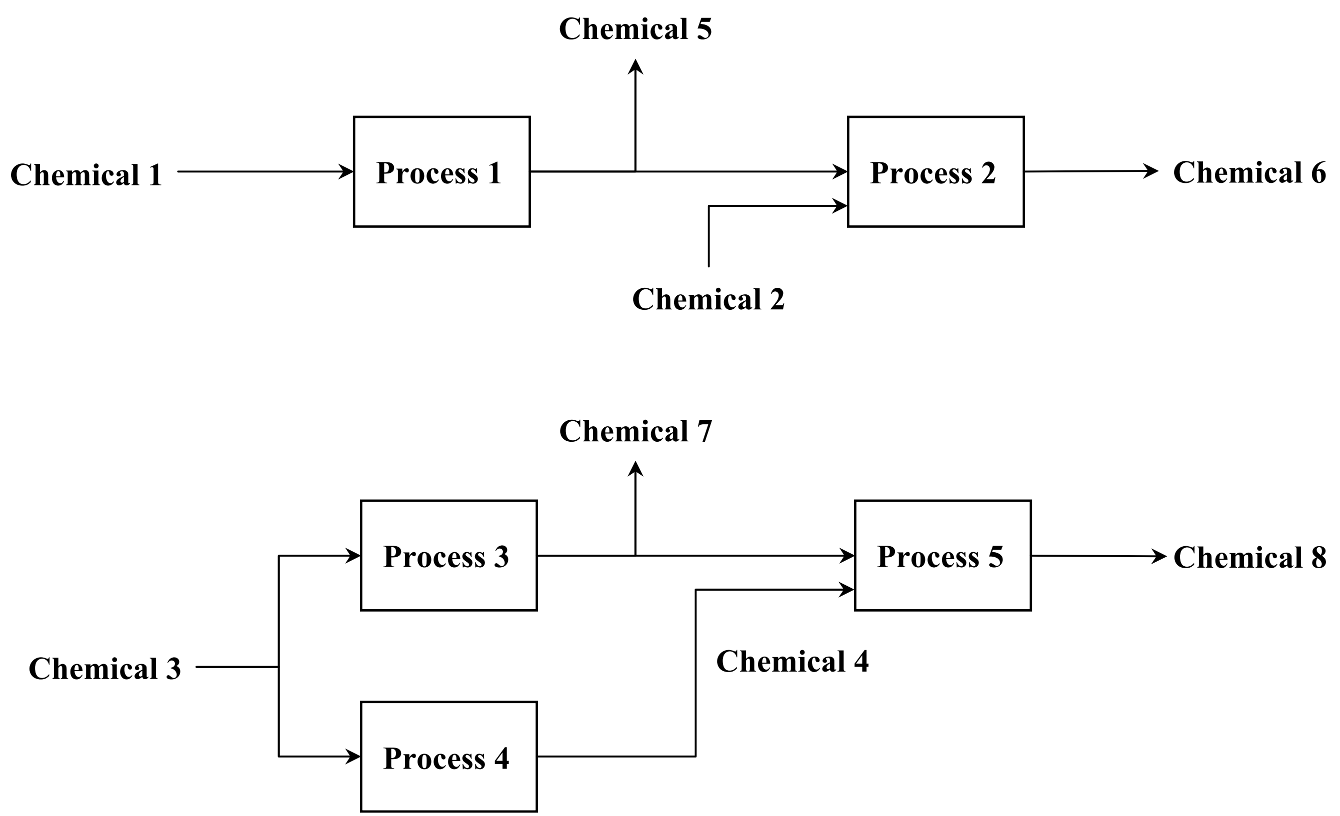
β_{it} : *Fixed cost of expansion for process i in period t*
L_t : *Discount factor for period t*

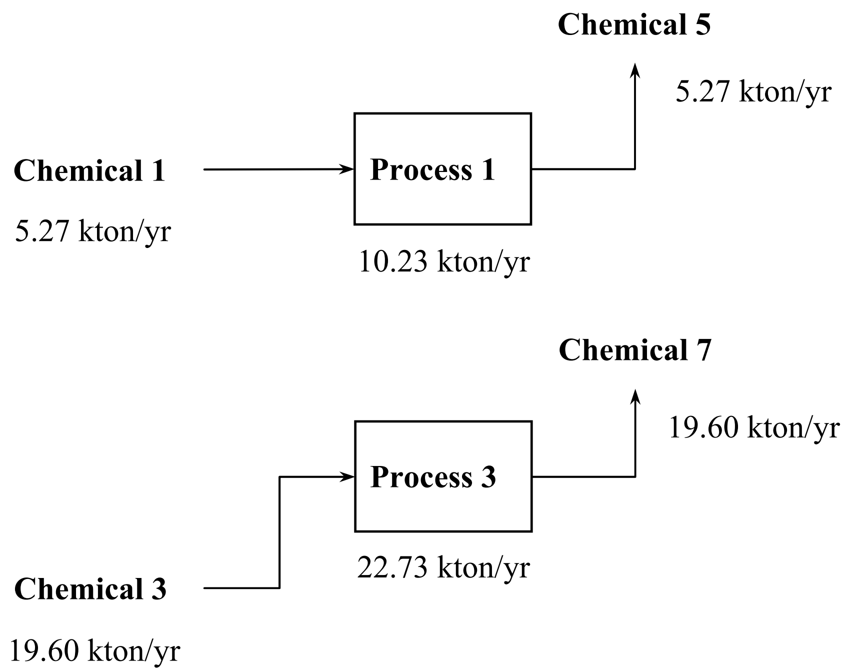
Example

- Uncertain Parameters: Demands, Availabilities, Sales Price, Purchase Price
- Total of 400 Scenarios
- Project Staged in 3 Time Periods of 2, 2.5, 3.5 years



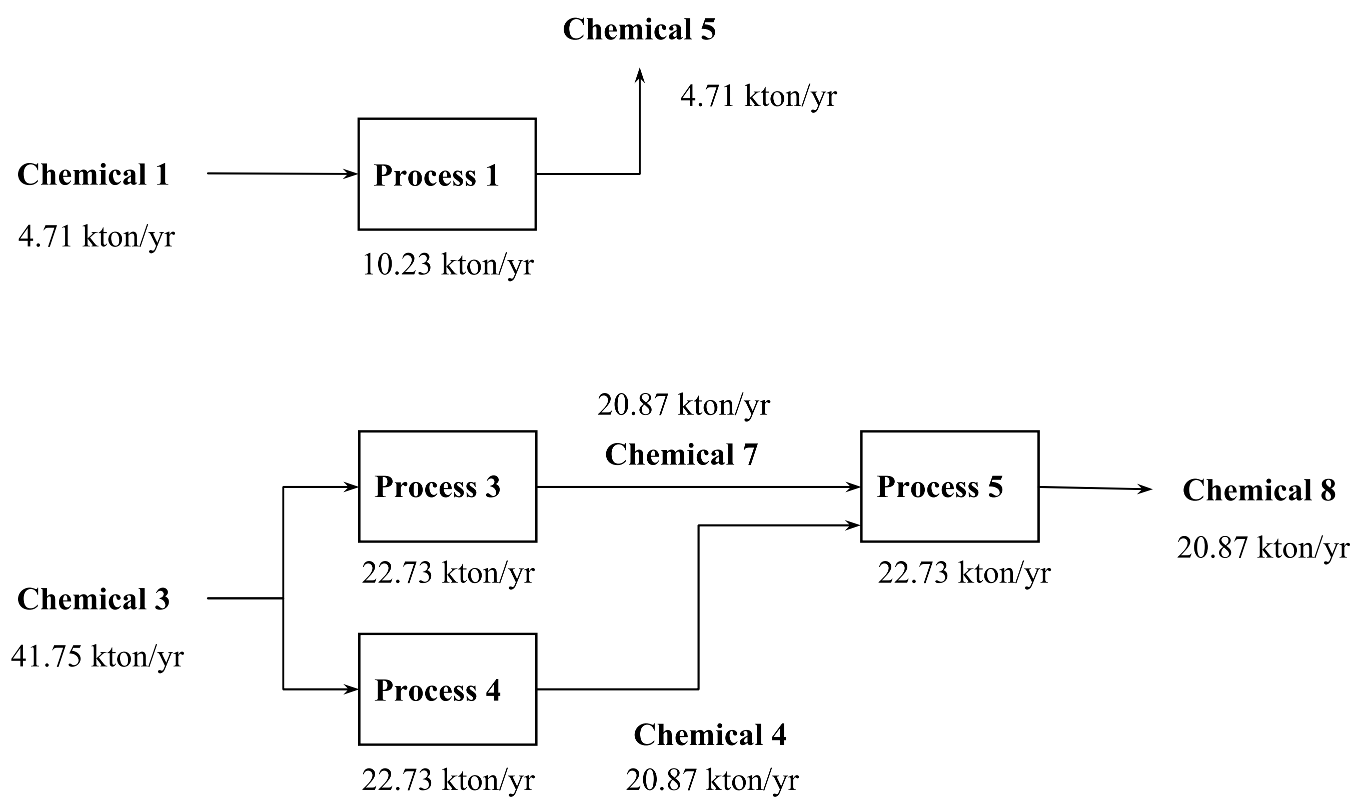
Example – Solution with Max ENPV

Period 1
2 years



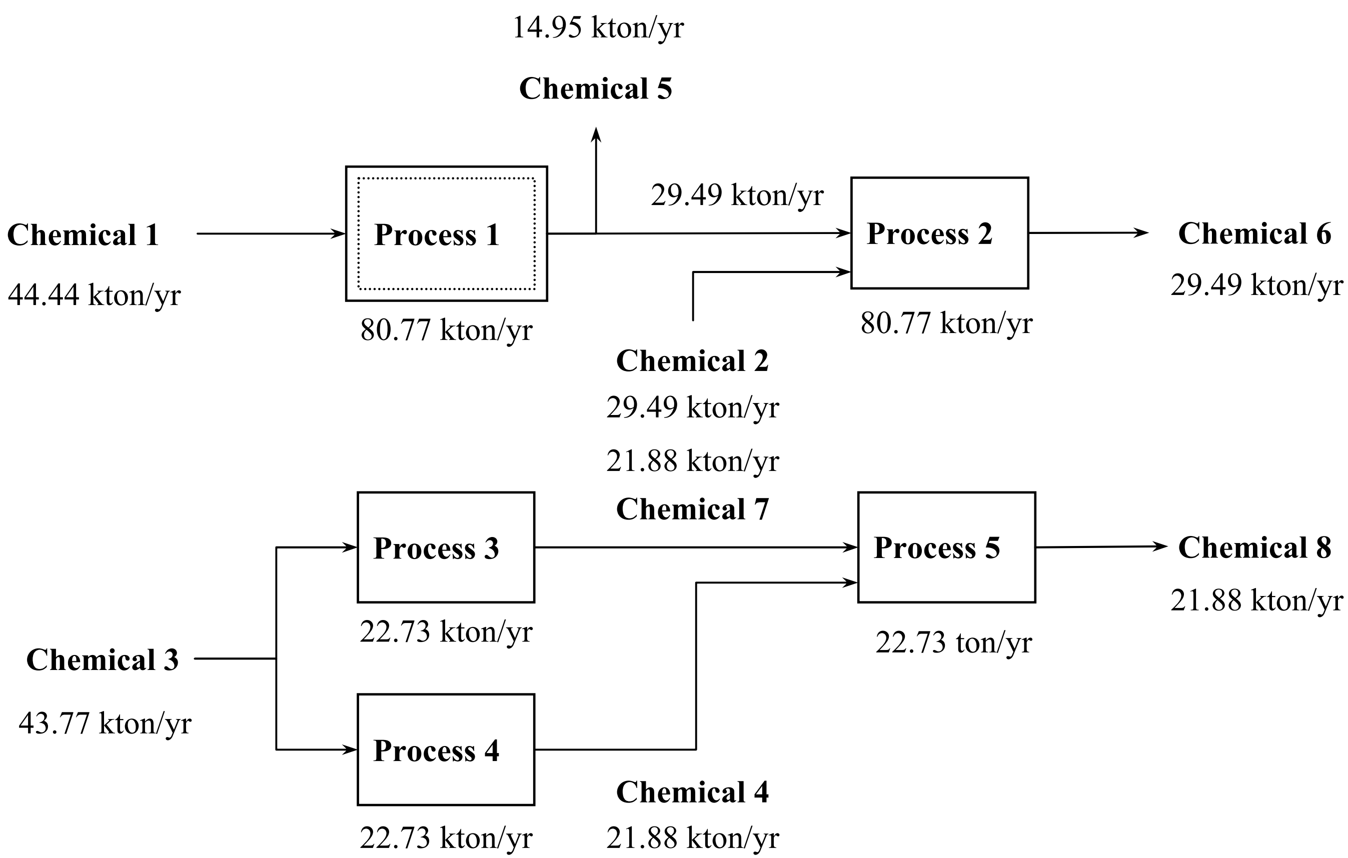
Example – Solution with Max ENPV

Period 2
2.5 years

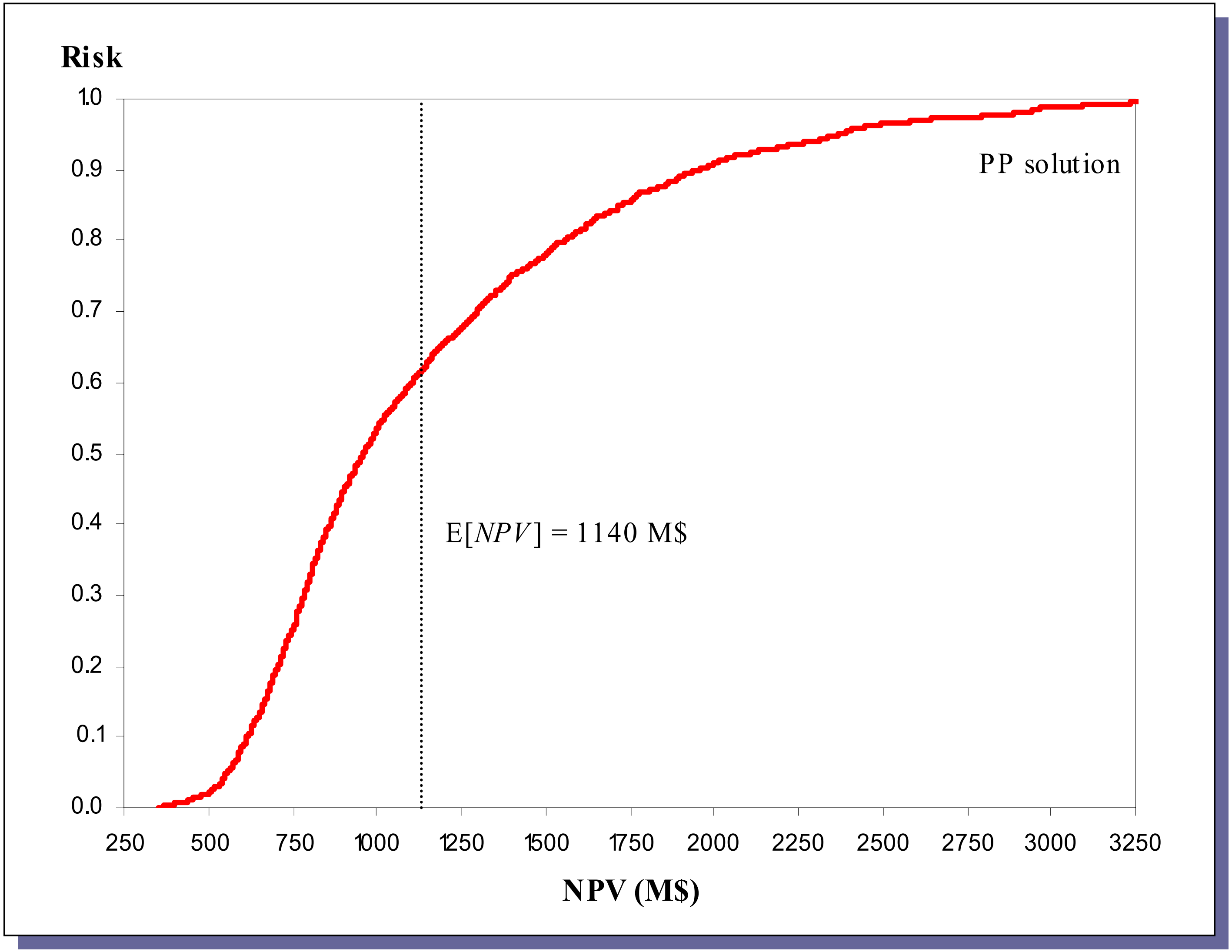


Example – Solution with Max ENPV

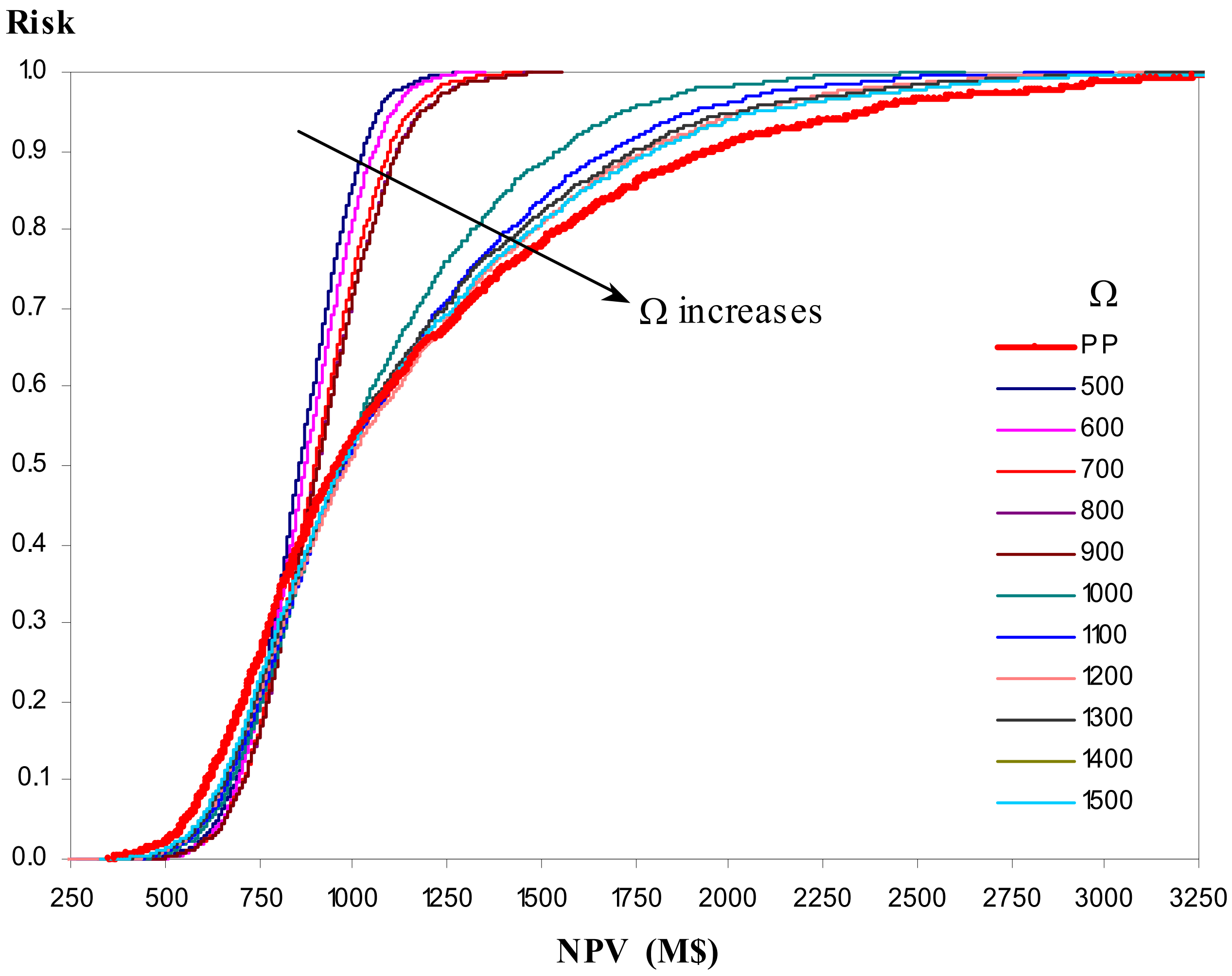
Period 3
3.5 years



Example – Solution with Max ENPV



Example – Risk Management Solutions



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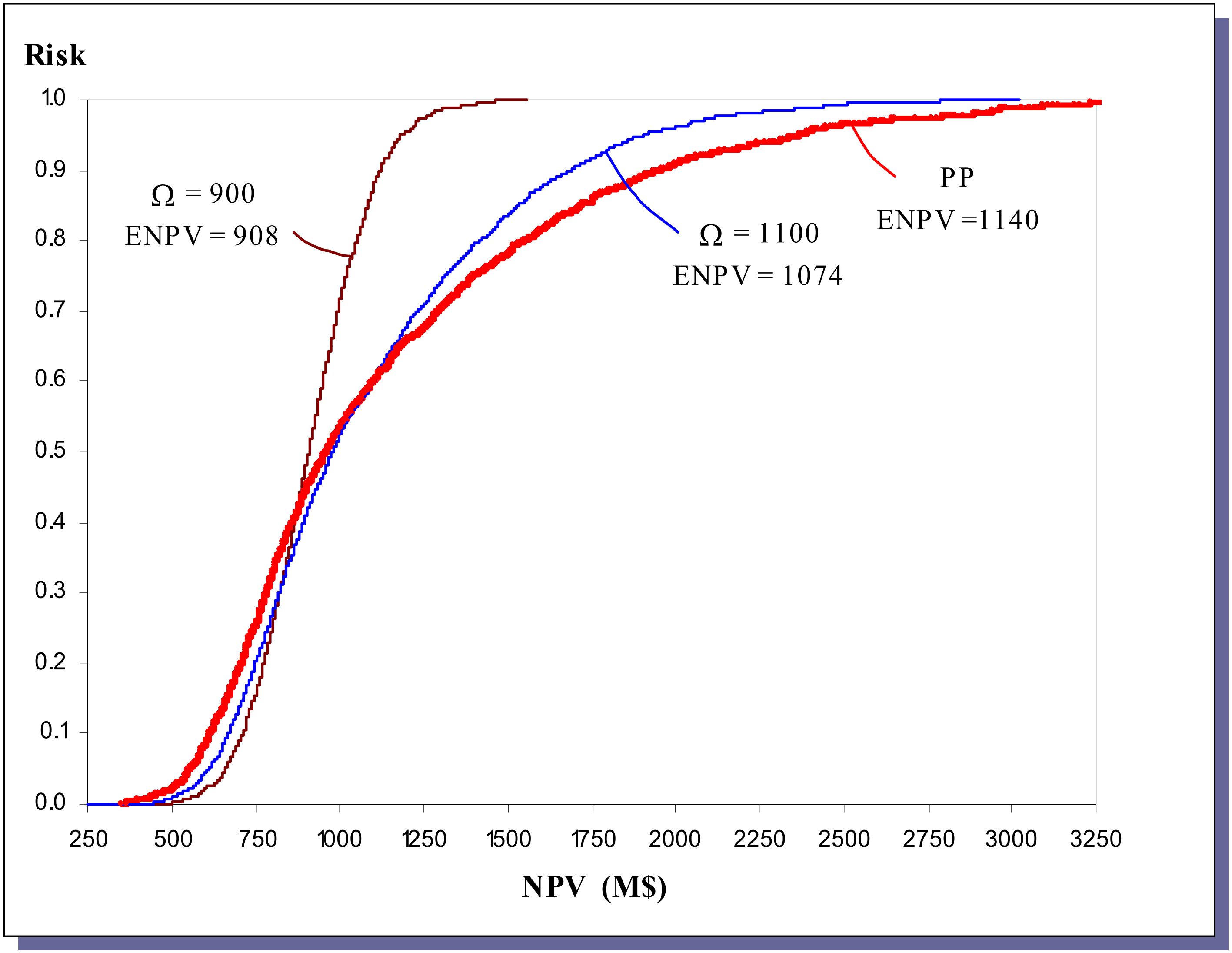

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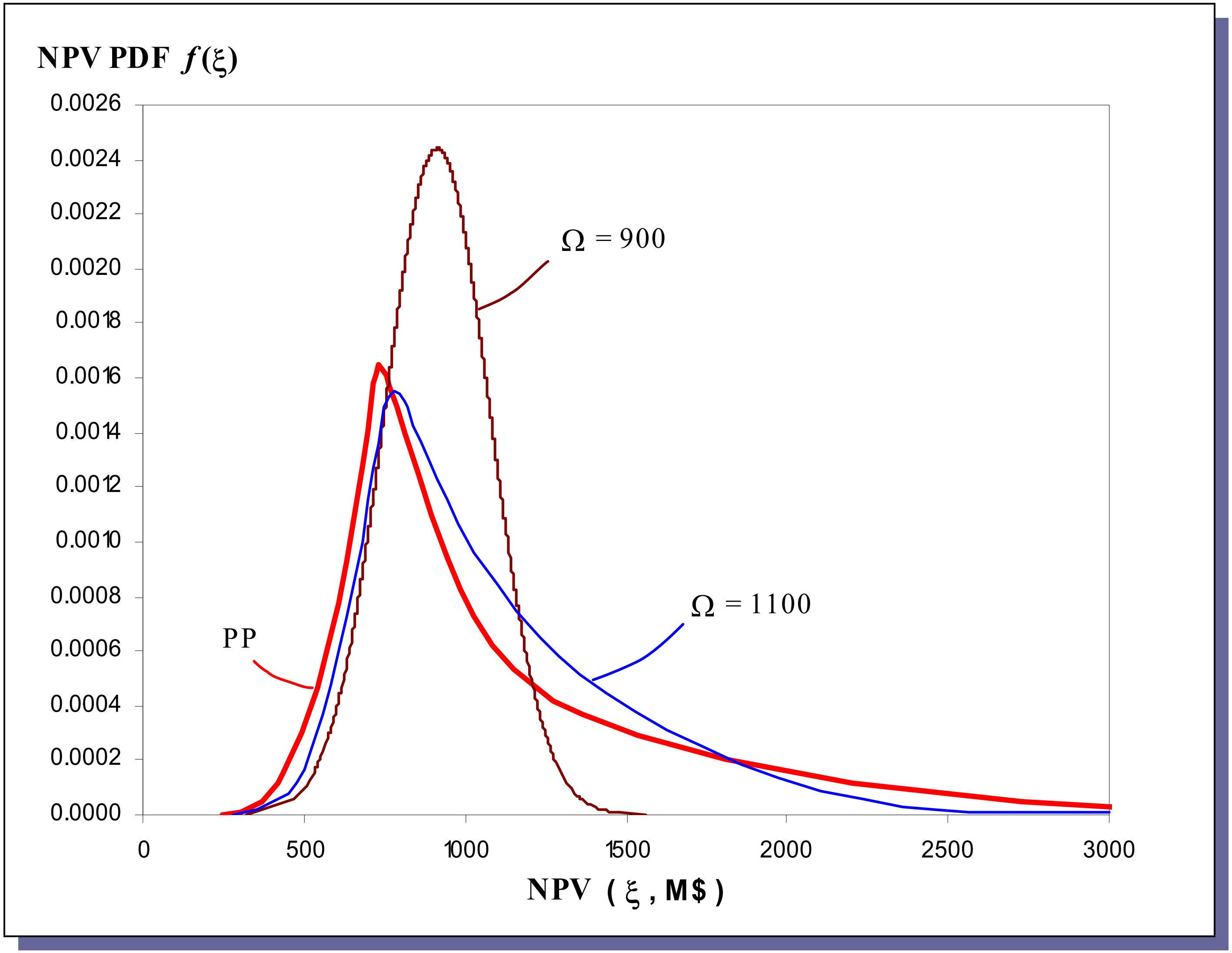
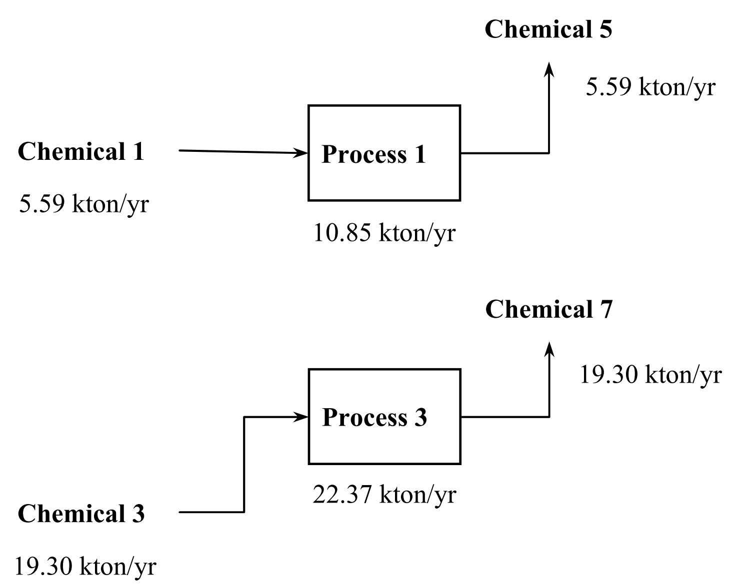


Example – Solution with Min DRisk(Ω=900)

Period 1
2 years

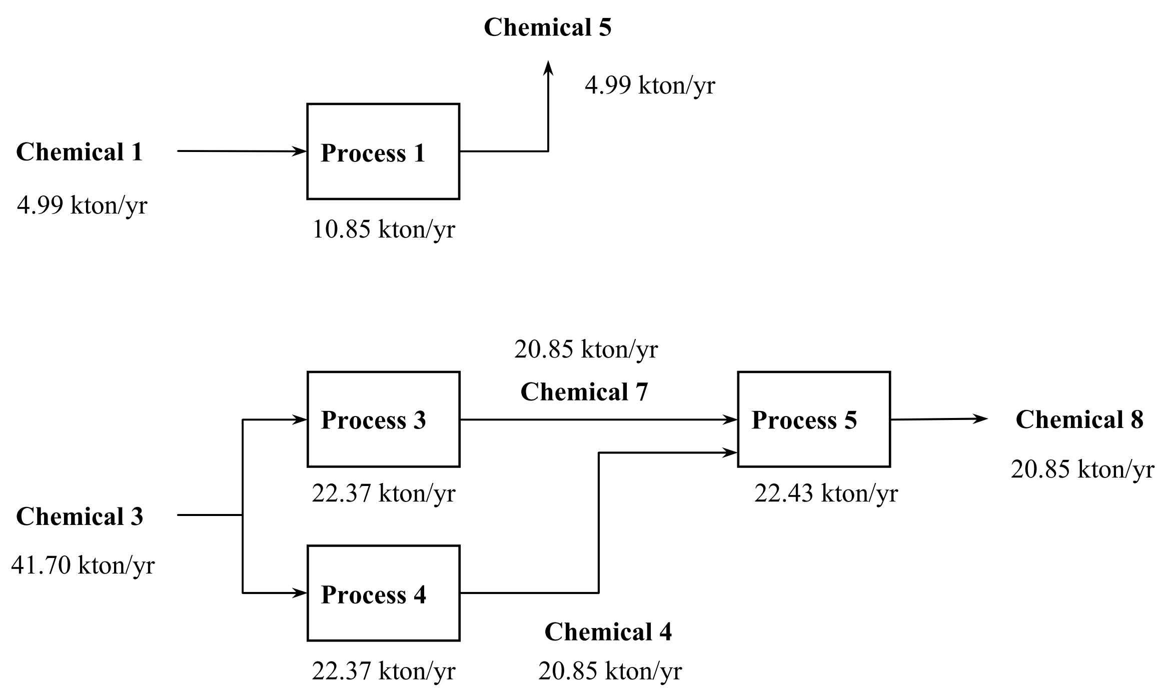
- **Same Flowsheet**
- **Different Capacities**



Example – Solution with Min DRisk(Ω=900)

Period 2
2.5 years

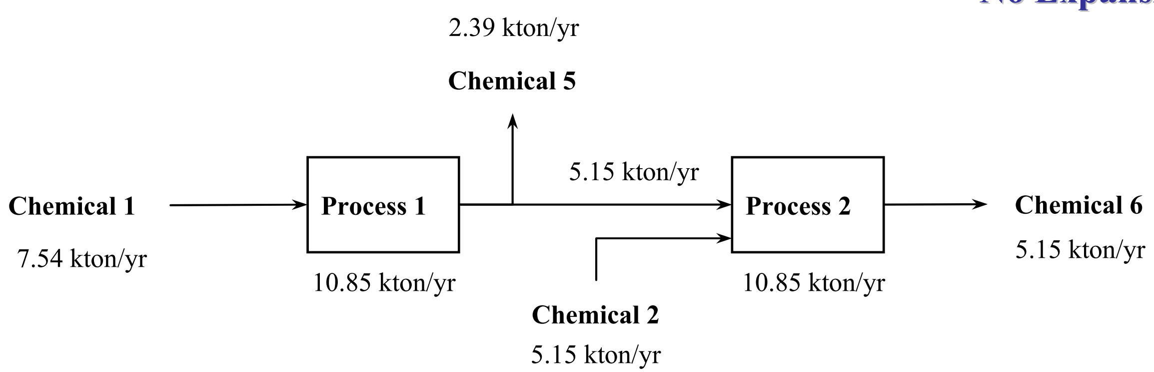
- Same Flowsheet
- Different Capacities

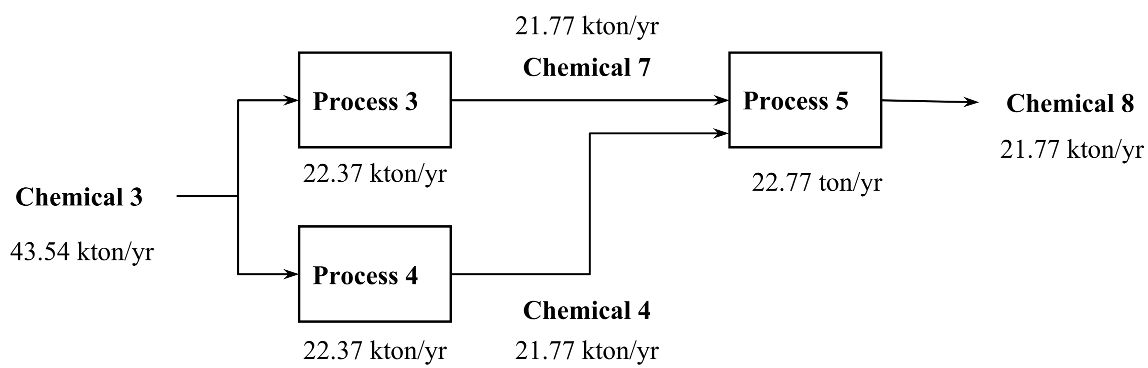


Example – Solution with Min DRisk(Ω=900)

Period 3
3.5 years

- **Same Flowsheet**
- **Different Capacities**
- **No Expansion of Process 1**

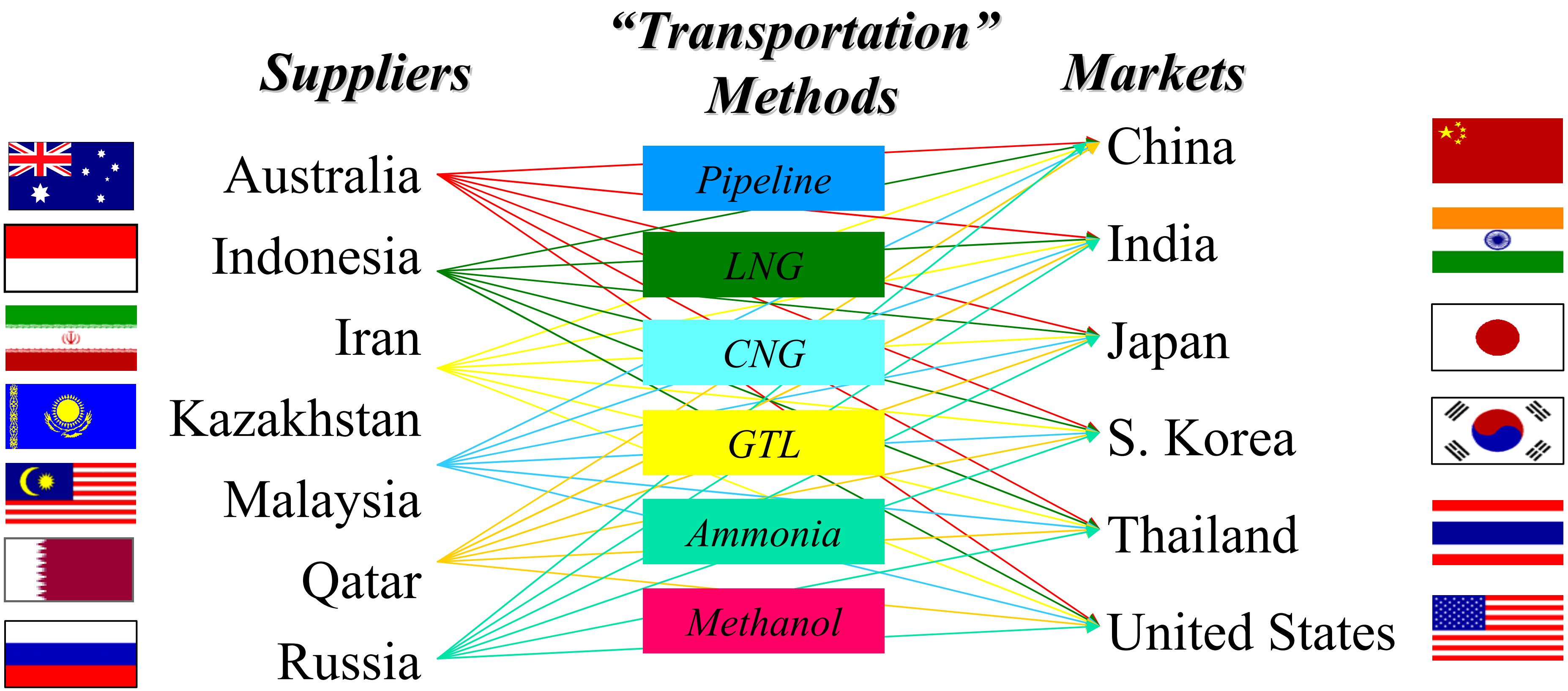




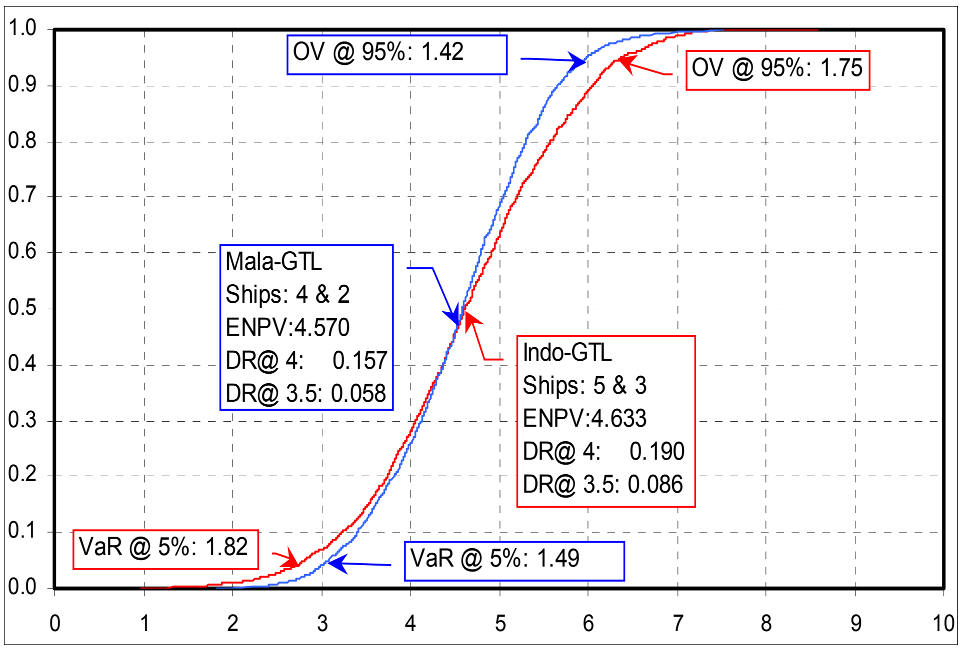
RECENT RESULTS

Gas Commercialization in Asia

Network of Alternatives



Some solutions



Time Period	FCI	Processing Facilities				Transportation to:				
		Mala (GTL)				China		Thai		Avrg. Ships
		Cap	Flow	Feed	Ships	Ships	Flow	Ships	Flow	
T1	3.00									
T2		4.57	4.47	297.9	4.0	1.16	0.98	2.79	3.49	3.95
T3	1.89	4.57	4.57	304.9	4.0			3.66	4.57	3.66
T4		7.49	7.32	488.2	6.0	0.42	0.35	5.58	6.97	6.00
T5		7.49	7.49	499.6	6.0			6.00	7.49	6.00
T6		7.49	7.49	499.6	6.0			6.00	7.49	6.00

Time Period	FCI	Processing Facilities				Transportation to:				
		Indo (GTL)				China		Thai		Avrg. Ships
		Cap	Flow	Feed	Ships	Ships	Flow	Ships	Flow	
T1	3.00									
T2		4.43	4.25	283.1	5.0	1.12	0.76	3.88	3.48	5.00
T3	1.90	4.43	4.43	295.5	5.0			4.94	4.43	4.94
T4		7.18	7.09	472.6	8.0	0.44	0.30	7.56	6.79	8.00
T5		7.18	7.18	479.0	8.0			8.00	7.18	8.00
T6		7.18	7.18	479.0	8.0			8.00	7.18	8.00

Conclusions

- Risk is usually assessed after a design/plan has been selected but it cannot be managed during the optimization stage (even when stochastic optimization including uncertainty has been performed).

- If Risk is to be managed, the decision maker has two simultaneous objectives:

  - Maximize Expected Profit.
  - Minimize Risk Exposure

- There are some good ways of obtaining good solutions (next lecture).