# Multiple Plant Heat Integration in a Total Site

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Heat integration across plants can be accomplished either directly using process streams or indirectly using intermediate fluids. Extensions to many plants of LP and MILP models previously developed for the two-plant case are presented. The targeting procedure first identifies the maximum possible savings for the whole system, and, subsequently, the minimum number of connections between the two-plant combinations required to obtain maximum savings is established. The location of intermediate fluid circuits to indirectly integrate the system is then found. Alternative solutions exist, which allow flexibility when the design of a multipurpose heat exchanger network is pursued. Finally, the optimal location of the circuits for cases of restricted operation is discussed.

# Introduction

Since the onset of heat integration as a tool for process synthesis, energy-saving methods have been developed for the design of energy-efficient individual plants. Heat integration across plants (that is, involving streams from different plants in a complex) has always been considered impractical for various reasons. Among the arguments used is the fact that plants are physically apart from each other and, because of this separation, pumping and piping costs are high. However, an even more powerful argument against integration was the fact that different plants have different startup and shutdown schedules. Therefore, if integration is done between two plants and one of the plants is put out of service, the other plant may have to resort to an alternative heat exchanger network to reach its target temperatures. Plants may also operate at different production rates departing from design conditions and needing additional exchangers to reach desired operating temperatures. All these discouraging aspects of the problem led practitioners and researchers to leave opportunities for heat integration between plants unexplored. Nevertheless, heat integration among plants is being used in practice and the aforementioned adverse issues can be sorted out.

Integration across plants can be accomplished either directly using process streams or indirectly using intermediate fluids, like steam or dowtherms. Total site integration is the name coined when referring to this complex problem. Early studies by Dhole and Linnhoff (1992) and Hui and Ahmad (1994) on total site heat integration helped to determine lev-

els of generation of steam to indirectly integrate different processes. Since the generation and use of steam has to be performed at a fixed temperature level, opportunities for integration are sometimes lost. Rodera and Bagajewicz (1999a) developed targeting procedures for direct and indirect integration in the special case of two plants and demonstrated the drawbacks of using steam as an intermediate fluid. Application of pinch analysis showed that the heat transfer effectively leading to savings occurs at temperature levels between the pinch points of both plants. In some other cases, however, heat transfer in the external regions is also required to attain maximum savings (assisted heat integration). The use of cascade diagrams for each plant allows for the detection of unassisted and assisted cases. Distinction between these two cases was overlooked by procedures that make use of combined grand composite curves (Dhole and Linnhoff, 1992), or methods developed to determine heat transfer between zones (Ahmad and Hui, 1991; Amidpour and Polley, 1997). In addition, Rodera and Bagajewicz (1999b,c, 2001) presented a methodology to design multipurpose heat exchanger networks that can realize these savings and function in the two scenarios, integrated and not integrated.

In this article, generalized mathematical models are presented that extend the results originally developed for two plants (Rodera and Bagajewicz, 1999a) to the case of multiple plants. Targeting procedures were succinctly presented by Rodera and Bagajewicz (1999d) and Bagajewicz and Rodera (2000). In this article, a complete substantiation and discussion of all the methodologies presented earlier are presented and, in addition, additional material is included. First, an LP model that considers all possible heat transfer among plants

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leading to savings is reviewed. This formulation identifies energy-saving targets for direct and indirect integration by determining the amounts of heat to be transferred within established temperature intervals. Then, an MILP model that makes use of these targets is introduced to establish the minimum number of connections between the two-plant combinations. For indirect integration, another MILP model is proposed that locates independent intermediate-fluid circuits. A reformulation of the model is offered to lower the computational burden. Finally, it is shown how to obtain the optimal location of circuits allowing flexibility of operation. Examples showing the different features of this approach are presented. The direct and indirect targets for the heat integration of an entire oil refinery comprised of seven process units are calculated, and the practical implementation issues related are discussed.

# **Targeting Models for Heat Integration**

Background material regarding the analysis of the maximum possible transferable heat for the case of two plants was presented by Rodera and Bagajewicz (1999a). The extension to a site consisting of a set of n plants was briefly introduced by Bagajewicz and Rodera (2000), who defined new concepts. In the latter article, it is pointed out that effective heat transfer, that is, heat transfer that leads to energy savings, takes place between pinch temperatures of the supplier and receiving plants. Moreover, it is shown that assisting heat transfer, that is, heat transfer outside the region between pinches performed to debottleneck heat between pinches, does not have to be transferred necessarily in the direction opposite to effective heat. To generalize these ideas, the concepts of effective-supplier plant and effective-receiver plant, as well as assisted plant and assisting plant, were introduced. Detailed examples were shown to illustrate this. In the following sections, the maximum energy savings model for the case of two plants is the starting point for developing the general model for *n* plants.

#### Maximum energy savings model for the two-plant case

A trans-shipment model was introduced by Rodera and Bagajewicz (1999a) to establish the amount of heat that can be transferred within each interval for the particular case of two plants. Figure 1 illustrates the notation for total heat amounts transferred in each region.

When considering energy minimization, the obvious objective function is the sum of all the heating utilities used ( $\delta_0^{I}$  +  $\delta_0^{\text{II}}$ ). This objective function, however, does not minimize the flow of assisted heat. Therefore, the objective function used for the two-plant model  $(\delta_0^{I} + \delta_m^{II})$  consists of minimizing the sum of the heating utility of plant 1 (the effective-receiver plant), and the cooling utility of plant 2 (the effective-supplier plant). While minimizing the heating utility of the receiver plant is clearly necessary, minimizing the cooling utility of the supplier plant is unnecessary in the absence of assisting heat. Adding the cooling utility to the objective function accomplishes the goal of reducing the assisting heat below both pinch temperatures to the minimum strictly necessary. At the same time, the minimization of the heating utility of plant 1 reduces the assisting heat to plant 2 above both pinch temperatures to the strictly necessary minimum.



Figure 1. Directions of heat transfer for the two-plants case.

The flow of heat between intervals of n plants can be generalized without much difficulty, from the equations of the model for two plants. However, the objective function needs to be revisited. The starting point is the use of an alternative objective function. Notice first that

$$\delta_o^{\rm I} = \hat{\delta}_o^{\rm I} + Q_A - Q_E \tag{1}$$

$$\delta_m^{\rm II} = \hat{\delta}_m^{\rm II} + Q_B - Q_E \tag{2}$$

Therefore, the following objective functions are equivalent

$$\operatorname{Min}\left(\delta_{0}^{\mathrm{I}} + \delta_{m}^{\mathrm{II}}\right) \Leftrightarrow \operatorname{Max}\left\{2 \cdot Q_{E} - \left(Q_{A} + Q_{B}\right)\right\}$$
(3)

In the alternative objective function, the total effective heat is counted twice, because savings are attained both on heating and on cooling utilities. The simultaneous maximization of the heat that effectively leads to savings and minimization of the assisting heat amounts are clearly achieved. The purpose of the assisting heat is to debottleneck the heat cascade of the corresponding assisted plant. Therefore, in an assisted heat integration case, the increase of effective heat in one unit is achieved by transferring exactly one unit of assisting heat. Sometimes, however, the plants need to be simultaneously debottlenecked by the transfer of assisting heat within both the regions above and below pinch temperatures, as Figure 2 illustrates.

In this example, the optimal case (Figure 2a) features  $Q_E = 19$ ,  $Q_A = 4$ , and  $Q_B = 3$ . The suboptimal case (Figure 2b) features  $Q_E = 18$ ,  $Q_A = 3$ , and  $Q_B = 2$ . The value of the objective function is clearly the same for both cases. Therefore, even though the objective function used by Rodera and Bagajewicz (1999a) fails to distinguish optimal from suboptimal solutions, the illustrative examples used in that article are correct. The special case illustrated in Figure 2 was not considered in the analysis. Indeed, only when a reduction of the effective heat transfer can be accompanied by the same

amount of reduction in assisting heat above and below both pinches does this objective function fail to identify the optimum. While the case presented is very special, it prompts revisiting the definition of the appropriate objective function.

To remedy the aforementioned shortcomings, we return to the sum of heating utilities objective function that in general for *n* plants is  $Min \sum_{\forall j \in P} \delta_0^j$ . A small mathematical manipulation shows that this objective function is equal to Max  $Q_E$  for the case of two plants. Thus, the problem that needs to be

solved for the case of two plants is

$$\mathbf{P1} = \operatorname{Min}\left(\delta_{0}^{\mathrm{I}} + \delta_{0}^{\mathrm{II}}\right) \\
\text{s.t.} \\
\delta_{0}^{\mathrm{I}} = \hat{\delta}_{0}^{\mathrm{I}} + Q_{A} - Q_{E} \\
\delta_{0}^{\mathrm{II}} = \hat{\delta}_{0}^{\mathrm{II}} - Q_{A} \\
\delta_{i}^{\mathrm{II}} = \delta_{i-1}^{\mathrm{II}} + q_{i}^{\mathrm{II}} - q_{i}^{A} \\
\delta_{i}^{\mathrm{II}} = \delta_{i-1}^{\mathrm{II}} + q_{i}^{\mathrm{II}} + q_{i}^{A} \\
\delta_{i}^{\mathrm{II}} = \delta_{i-1}^{\mathrm{II}} + q_{i}^{\mathrm{II}} + q_{i}^{E} \\
\delta_{i}^{\mathrm{II}} = \delta_{i-1}^{\mathrm{II}} + q_{i}^{\mathrm{II}} - q_{i}^{E} \\
\delta_{i}^{\mathrm{II}} = \delta_{i-1}^{\mathrm{II}} + q_{i}^{\mathrm{II}} - q_{i}^{B} \\
\delta_{i}^{\mathrm{II}} = \delta_{i-1}^{\mathrm{II}} + q_{i}^{\mathrm{II}} - q_{i}^{B} \\
\delta_{i}^{\mathrm{II}} = \delta_{i-1}^{\mathrm{II}} + q_{i}^{\mathrm{II}} - q_{i}^{B} \\
\delta_{i}^{\mathrm{II}} = \delta_{i-1}^{\mathrm{II}} + q_{i}^{\mathrm{II}} + q_{i}^{B} \\
\delta_{i}^{\mathrm{II}} = \delta_{m}^{\mathrm{II}} - Q_{B} \\
\delta_{m}^{\mathrm{II}} = \hat{\delta}_{m}^{\mathrm{II}} + Q_{B} - Q_{E} \\
\delta_{i}^{\mathrm{II}}, \delta_{i}^{\mathrm{II}}, q_{i}^{A}, q_{i}^{E}, q_{i}^{B} \ge 0
\end{aligned}$$

$$(4)$$

The only problem with this objective function is that it is invariant to the value of the assisting heat above the threshold established by the value that allows the debottlenecking of the cascade. For example, take the solution shown in Figure 2a and increase the assisting heat. There is no effect on the value of the total heating utility  $(\delta_0^{I} + \delta_0^{II})$ . However, since  $Q_A = 4$  is the threshold, a reduction of  $Q_A$  below this value affects the total energy consumption. To fix the value of assisting heat to the minimum, a new problem needs to be solved. Let  $Q_E^*$  be the optimal value of effective heat transfer between the two plants as determined using problem **P1**. Then, the following problem minimizes the assisting heat above and below both pinch temperatures

$$\begin{array}{c} \mathbf{P2} = \operatorname{Min} \left( Q_A + Q_B \right) \\ \text{s.t.} \\ Q_E = Q_E^* \\ \text{All constraints of problem P1} \end{array}$$

$$(5)$$

A penalty function version for P2 is

$$\mathbf{P3} = \operatorname{Min} \left\{ (Q_A + Q_B) + \mu f(Q_E) \right\}$$
  
s.t.  
All constraints of problem **P1** (6)

Because  $Q_E^* \ge Q_E$ , then the penalty function for **P2** is linear, that is,  $f(Q_E) = (Q_E^* - Q_E)$ , and because  $Q_E^*$  is a constant, it can be dropped. Therefore, the problem can be rewritten as

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PLANT 2 **PLANT 1** 16 = 20 - 47 = 22 + 4 - 19-7 ·10 0 = 7 - 76 = 16 - 10Pinch 0 = 6 - 10 + 414 0 = 1 - 15 + 140 = 14 - 14-5 7 Pinch 2 = 0 + 7 - 57 5 = 0 + 7 - 3= 2 - 5 + 33 2 7 = 4 + 3(a) Optimal case PLANT 1 PLANT 2 7 = 22 + 3 - 1817 = 20 - 30 = 7 - 77 = 17 - 105 3 10 Pinch 2 = 0 + 5 - 30 = 7 - 10 + 314 1 = 2 - 15 + 140 = 14 - 147 -5 Pinch 0 = 13 = 03 2 8 = 5 + 32 = 0 + 2(b) Sub-optimal case

Figure 2. Cascade example of simultaneous assisted heat.

follows

$$\mathbf{P3'} = \operatorname{Max} \left\{ Q_E - \epsilon (Q_A + Q_B) \right\}$$
  
s.t.  
All constraints of problem **P1** (7)

where  $\epsilon = 1/\mu$ . Thus, if the proper value of  $\mu$  is used, the solution of problem **P3** is the same as the solution obtained solving **P1** and **P2** in sequence. The issue is then to determine the proper value of  $\mu$ . The answer is  $\mu > 2$ . Indeed, as can be seen from the example of Figure 2, when  $\mu = 2$ , all cases, except the ones that require double assistance (above and below both pinch temperatures simultaneously), will ren-

der the correct optimal solution. In the special case illustrated in Figure 2, the problem is degenerate, as was illustrated above. Thus, for  $\mu > 2$ , the effective heat, which always produces two units of total savings per unit of assisting heat, will have a larger weight than the assisting heat. This very same analysis can be made for the case of multiple plants, as the worst case scenario is that a certain amount of effective heat can be transferred only if a debottlenecking takes place in the effective supplier and receiver plants. The only difference is that the assisted heat now can be provided by any plant.

#### Maximum energy savings model for the total-site

The model that considers independent transfer of heat within each interval is presented next. This model accounts for transfer from effective-suppliers to effective-receivers between pinch temperatures, from assisted to assisting plants above their pinch temperatures, and from assisting to assisted plants below their pinch temperatures. The definition for the sets was introduced by Bagajewicz and Rodera (2000), and it is reproduced in the notation section. The model follows

$$\begin{aligned} \mathbf{P4} &= \operatorname{Max} \left\{ Q_{E} - \epsilon \left( Q_{A} + Q_{B} \right) \right\} \\ \text{s.t.} \\ Q_{E} &= \sum_{j \in P} \sum_{k \in S_{j}^{E}} Q_{kj}^{E} \\ Q_{A} &= \sum_{j \in P} \sum_{k \in R_{j}^{A}} Q_{jk}^{A} \\ Q_{B} &= \sum_{j \in P} \sum_{k \in R_{j}^{B}} Q_{jk}^{B} \\ Q_{kj}^{E} &= \sum_{i = p^{k} + 1, \dots, p^{i}} q_{ijk}^{E} \quad k \in S_{j}^{E} \\ Q_{jk}^{A} &= \sum_{i = p^{i} + 1, \dots, p^{i}} q_{ijk}^{B} \quad k \in R_{j}^{A} \\ Q_{jk}^{B} &= \sum_{i = p^{i} + 1, \dots, m^{i}} q_{ijk}^{B} \quad k \in R_{j}^{A} \\ Q_{jk}^{B} &= \sum_{i = p^{i} + 1, \dots, m^{i}} q_{ijk}^{B} \quad k \in R_{j}^{A} \\ \delta_{o}^{j} &= \delta_{o}^{j} - \sum_{k \in S_{j}^{E}} Q_{kj}^{E} + \sum_{k \in R_{j}^{A}} Q_{jk}^{A} - \sum_{k \in S_{j}^{A}} Q_{kj}^{A} \\ \delta_{i}^{j} &= \delta_{i-1}^{j} + q_{i}^{j} + \sum_{k \in S_{ij}^{E}} q_{ikj}^{E} - \sum_{k \in R_{ij}^{A}} q_{ijk}^{A} \\ + \sum_{k \in S_{ij}^{A}} q_{ikj}^{A} \qquad \forall i = 1, \dots, p^{j} \\ \delta_{i}^{j} &= \delta_{i-1}^{j} + q_{i}^{j} - \sum_{k \in R_{ij}^{E}} q_{ijk}^{E} \forall i = (p^{j} + 1), \dots, m \\ \delta_{m}^{j} &= \delta_{m}^{j} - \sum_{k \in R_{ij}^{E}} Q_{jk}^{E} + \sum_{k \in S_{j}^{B}} Q_{kj}^{B} - \sum_{k \in R_{ij}^{B}} Q_{jk}^{B} \\ \delta_{j}^{j}, q_{ijk}^{A}, q_{ikj}^{E}, q_{ijk}^{B} \ge 0 \end{aligned}$$

where the value of  $\epsilon$  is smaller than 0.5, as determined by the analysis of the previous section. The overall effective heat-transfer amount  $Q_E$  and the eventual assisted heat amounts  $Q_A$  and  $Q_B$  are the summation of the corresponding heat amounts transferred between all the pair combinations. These overall amounts of heat transferred between plants  $Q_{kj}^E, Q_{jk}^A$ , and  $Q_{jk}^B$  are related to the heat transferred amounts in each interval  $q_{ikj}^E, q_{ijk}^A$ , and  $q_{ijk}^B$ , as found by simple addition. Finally, the model contains the well-known cascade heat balance equations.

In its original form for the case of two plants (Rodera and Bagajewicz, 1999a), indirect integration and particularly the difficulty of having intermediate circuits transferring heat in different directions was solved by shifting the temperature scales of the plants. These temperature scale shifts cannot be applied to more than two plants, as the multiple shifts conflict with each other. Therefore, in these models, indirect integration (and, particularly, the issue of having intermediate circuits transferring heat in different directions) is addressed considering that the variables representing heat transfer between plants correspond to upward and downward diagonal transfer. This diagonal transfer is established between intervals of the same length that are located a fixed number of intervals apart. The procedure used for obtaining this generalized structure of intervals is given in Appendix A.

#### Minimum number of connections

The result obtained by solving model **P4** represents the maximum possible savings for the whole system. The solution, however, does not control the number of required connections between plant pairs and may include too many connections. In other words, the problem is degenerate, and an additional step is needed to choose the appropriate connections. The following model introduces three different sets of binary variables  $(X_{kj}^E, X_{jk}^A, X_{jk}^B)$  to account for connections from effective-supplier plants to effective-receiver plants between pinch temperatures, from assisted to assisting plants above their pinch temperatures.

$$\mathbf{P5} = \operatorname{Min} \sum_{j \in P} \left( \sum_{k \in S_{j}^{E}} X_{kj}^{E} + \sum_{k \in R_{j}^{A}} X_{jk}^{A} + \sum_{k \in R_{j}^{B}} X_{jk}^{B} \right)$$
s.t.  

$$Q_{E} = Q_{E}^{*}$$

$$Q_{A} + Q_{B} = Q_{A}^{*} + Q_{B}^{*}$$
All constraints of problem **P4**  

$$L_{kj}^{E} X_{kj}^{E} \leq Q_{kj}^{E} \leq U_{kj}^{E} X_{kj}^{E} \quad k \in S_{j}^{E}$$

$$L_{jk}^{A} X_{jk}^{A} \leq Q_{jk}^{A} \leq U_{jk}^{A} X_{jk}^{A} \quad k \in R_{j}^{A}$$

$$J_{jk}^{E} X_{jk}^{B} \leq Q_{jk}^{B} \leq U_{jk}^{B} X_{jk}^{B} \quad k \in R_{j}^{B}$$

$$X_{kj}^{E}, X_{jk}^{A}, X_{jk}^{B} \in \{0,1\}.$$
(9)

The targeted amounts of effective and assisting heat are used to fix the total heat transferred. Notice that, although assisting heat is represented with two independent variables for amounts of heat transferred above and below the pinch

Table 1. Individual Plant Pinch Analysis forExample 1

Problem	Pinch Temp. (°C)	Min. Heating Utility (kW)	Min. Cooling Utility (kW)
Area A	70	43	12.75
Area B	500	1.5	19.35
Area C	210	25	36.5

temperatures, the total amount of assisting heat determined by the summation of the separate targets is maintained. This allows the separate targets to be rearranged. Finally, additional constraints provide upper and lower bounds for the connections.

The purpose of the model is to find the minimum number of connections in order to attain maximum effective savings. However, the relaxation of the value of total effective heat transfer may lead to a reduction in the number of connections. Moreover, values of assisting heat may be obtained that also reduce the number of connections. It is clear that a trade-off between energy savings and the number of connections exists. Additionally, in the above model, all connections are considered to have the same length, hence, the objective, which minimizes indirectly the fixed cost associated with units exchanging heat between different plants. If one wants to factor in the influence of piping costs, one can add weights



Figure 3. Comparison between targeting approaches for Example 1.



Figure 4. Minimum number of heat flows for Example 1.

proportional to distances between plants. Thus, for the same level of heat recovery, the minimum distance between plants can be chosen. Numerically, this represents no additional difficulty. The concepts developed up to this point are now illustrated.

#### Example 1

This example, introduced by Ahmad and Hui (1991), is used here to pinpoint the differences between their procedure and our proposed targeting approach. Table 1 shows the results of independently applying pinch analysis to each of the "areas of integrity" that can be considered individual plants.

Figure 3a shows the solution obtained by solving problem **P4**. Both types of assisted heat integration (that is, in the opposite direction and in the same direction as the effective heat) take place between area C and area B. These assisting heats allow effective savings, not only between these areas, but also between area C and area A, while the system reaches maximum savings. Figure 3b illustrates the required flows and duties between the same areas. This solution can be found using the procedure suggested by Ahmad and Hui (1991). However, to do so, the appropriate choices of heats to be disallowed must be made (that is, no automatic solution is possible), and this requires some *a priori* knowledge. Moreover, their procedure overlooks the insights gained by considering effective heat transfer and assisting heat transfer between plant regions. When model **P5** is solved, four connec-



Figure 5. Minimum number of heat flows for Example 1 (alternative solution).

tions are obtained, and the interval heat transfers have the same values, as seen in the solution of problem **P4**. Two connections transfer effective heat from area B to area C and from area C to area A, and two connections transfer assisting heat below pinch temperatures from area B to area C, and vice versa.

The reported solution to the procedure which finds the required heat flows between the regions (Ahmad and Hui, 1991), however, results in the scheme shown in Figure 4b. Note that this solution requires one less flow between the areas (no flow from area B to area A exists). Nevertheless, Figure 4a proves that this solution can be found by restricting the flow from area B to area A in problem **P4**. Notice that if effective and assisting connections are separately considered, the result is an alternative solution to the one presented in Figure 3a. As this flow of 24 units can be separated from the assisted flow of 1.45 units (because they are transferred in different regions), the number of connections is again four (that is, two effective and two assisting below pinch temperatures). Another alternative solution is shown in Figure 5.

We conclude that targets can be obtained automatically by solving problem **P4**. Moreover, a distinction between the effective and assisting heat flows between areas is made using this model. On the other hand, the procedure presented by Ahmad and Hui (1991) requires an iterative procedure and some decision-making. Total heat flows between the areas are obtained without differentiating between effective and

 Table 2. Individual Plant Pinch Analysis for

 Example 2

Problem	Pinch Temp. (°C)	Min. Heating Utility (kW)	Min. Cooling Utility (kW)
Test Case No. 2	90	107.5	40.0
Trivedi (1988)	160	404.8	688.6
Ciric and Floudas (1991)	200	600.0	2,100.0
4sp1	249	128.0	250.0

assisting heat transfer. The solution to problem **P5** will automatically determine the number of connections that distinguish between effective and assisting connections.

# Example 2

This example was constructed using a combination of Examples 4 and 5 from Rodera and Bagajewicz (1999a). It was used in Bagajewicz and Rodera (2000) to show the integration among a set of four plants. The results of applying individual pinch analysis to each of the plants are shown in Table 2.

# **Direct** integration

The results of applying direct heat integration to this example by solving problem **P4** are shown in Figure 6, which is taken from Bagajewicz and Rodera (2000). This is an instance of assisted heat integration (heat is sent from plant 2 to plant 3 to debottleneck the heat cascade of plant 2).

Table 3 shows the amount of savings achieved in each of the plants, as well as the maximum savings for the system.

One of the alternative solutions for the direct heat integration case is shown in Figure 7. This solution was obtained by first disallowing heat transfer between plant 1 and plant 2 and then by solving problem **P4**.



Figure 6. Direct integration solution for Example 2.

Table 3. Indirect Integration for Example 2

	Savings (kW)		
Problem	Heating	Cooling	
Test Case No. 2	107.5	0.0	
Trivedi (1988)	149.9	107.5	
Ciric and Floudas (1991)	173.6	104.5	
4sp1	0.0	219.0	
Total savings	431.0	431.0	



Figure 7. Alternative direct integration solution for Example 2.

Table 4 shows the amount of savings for this case. The total amounts are equal to those in the alternative presented in Table 3. However, the individual savings reflect the different characteristics of this alternative.

#### Indirect integration

When indirect integration is explored for this example, a lower amount of savings than in the direct integration case is observed. The region leading to effective savings is reduced, because diagonal transference between equal intervals is required in order to use an intermediate fluid. The solution for the assisted indirect heat integration after solving problem

	PLANT 1	PLANT 2	PLANT 3	PLANT 4
(1	Fest Case #2)	(Trivedi)	(C & F)	(4sp1)
	$T_0$	<b>J</b> 353.2	J 392.4	$\int 128$
<sup>300</sup> °C⊤				
249 °C-				
			June 17	
200 °C		33.3		
				-7///
160 °C				
90 °C−				
40 °C	Ļ	ЦJ		
	<b>↓</b> <sup>40</sup>	<b>↓</b> <sup>688.6</sup>	1983.4	<b>↓</b> <sup>0</sup>

Figure 8. Indirect integration solution for Example 2.

P4 is shown in Figure 8 taken from Bagajewicz and Rodera (2000). Notice that, although Plant 3 continues to assist Plant 2, the pattern of effective heat transfer changes with respect to direct heat integration.

Table 5 shows the amount of savings achieved in each of the plants, as well as the maximum savings achieved when the system is indirect heat integrated.

The indirect integration solution presented does not consider the minimization of the number of effective and assisting connections between plants. A solution featuring the minimum number of connections is obtained by solving problem P5 and is shown in Figure 9. This alternative solution contains only four connections: three effective connections and one assisting connection from plant 2 to plant 3.

The amount of savings is equal to those in the alternative previously presented. However, the individual savings reflect the unique characteristics of this alternative (Table 6).

#### Example 3

To test the developed tools in a large and realistic problem, the heat integration between seven units of an entire oil refinery is considered. The data for these units can be found in Fraser and Gillespie (1992) who applied pinch technology to energy integrate the whole system. The results reported by these authors are based on current plant heating utility usage

Table 5. Indirect Integration for Example 2

Fable 4.	Direct Integration for Example 1 (Alternative	
	Solution)	

Heating

107.5

51.6

0.0

271.9

431.0

Savings (kW)

Coolir

)		Savings (kW)	
Cooling	Problem	Heating	Cooling
0.0	Test Case No. 2	107.5	0.0
0.0	Trivedi (1988)	51.6	0.0
180.9	Ciric and Floudas (1991)	207.5	116.6
250.1	4sp1	0.0	250.1
431.0	Total savings	366.7	366.7

4sp1

Problem

Ciric and Floudas (1991)

Test Case No. 2

Trivedi (1988)

Total savings



Figure 9. Indirect integration solution for Example 2 (minimum number of connections).

(none of the existing plants is completely energy integrated). Since their savings have a retrofit component, comparisons with their approach will not be made. Table 7 shows the results of applying pinch analysis to the individual plants.

#### **Direct** integration

When problem P4 is solved, effective-direct integration savings of 19.47 MW are obtained. This represents a 23% savings of the total heating utility of the site. An amount of 3.30 MW of assisted heat is required to be transferred above pinch temperatures, and an amount of 2.34 MW below pinch temperatures. These targeting values are then used to formulate problem P5, which is solved obtaining the minimum number of connections and their respective heat fluxes. Figure 10 shows the solution containing 17 connections, eleven of which are the effective connections leading to savings. The rest are assisting connections in the opposite direction to the effective connections. Table 8 shows what the amount of savings achieved in each of the units are, as well as in the entire oil refinery. The excessive amount of inter-unit connections required to attain maximum savings makes questionable the practicality of this solution.

# Table 6. Indirect Integration for Example 2 (Minimum Number of Connections)

	Savings (kW)	
Problem	Heating	Cooling
Test Case No. 2	107.5	0.0
Trivedi (1988)	51.6	107.5
Ciric and Floudas (1991)	207.6	84.9
4sp1	0.0	174.3
Total savings	366.7	366.7

#### Indirect integration

After the interval partition procedure given in Appendix A is performed and indirect diagonal transfer is considered, problem **P4** is solved. Effective-indirect integration savings of 18.03 MW are obtained, which represents a 21% savings of the total heating utility of the site. An amount of 3.3 MW of assisted heat is required to be transferred above pinch temperatures, and an amount of 2.3 MW is required below pinch temperatures. The minimum number of connections and their respective heat fluxes are then obtained by solving problem **P5** with the use of these targeting values. Figure 11 shows the solution containing 20 connections, 14 of which are the effective connections in the opposite direction to the effective connections. Table 9 shows the amount of savings achieved in each of the units, as well as the entire oil refinery.

As in direct integration, the excessive amount of inter-unit connections required to attain maximum savings raises the question of the practicality of this solution. A procedure to sequentially increment the number of targeting circuits and their location is presented next.

#### **Targeting Model for Circuits Location**

The location of the minimum number of independent intermediate-fluid circuits necessary to attain maximum savings for indirect integration is found by a procedure equivalent to the step-by-step increase in the number of circuits that was presented by Rodera and Bagajewicz (1999a) for the special case of two plants. The procedure starts by solving an extension to the total site of the MILP model that was developed to find the location of a single intermediate-fluid circuit (Rodera and Bagajewicz, 1999a). Then, the number of independent circuits is increased until the maximum possible savings for indirect integration is attained. The model finds the best arrangement for a particular number of circuits in the sequence by considering all possible plant-pair combinations.

Unit	Pinch Temp. (°C)	Min. Heating Utility (MW)	Min. Cooling Utility (MW)
Platformer (reformer)	79.4	18.00	8.37
Visbreaker (thermal cracking)	145	6.83	3.20
Kerosene hydrotreater	176.6	0.73	4.19
Naphtha hydrotreater	177.2	4.17	7.73
Crude and vacuum distillation	272	54.94	23.94
Fluid-catalytic cracking		0.00	20.45
Diesel hydrotreater		0.00	2.76
Entire refinery	NA	84.67	70.64

Table 7. Individual Plant Pinch Analysis for Example 3

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$$\begin{aligned} \mathbf{P6} &= \operatorname{Max} Q_{E} \\ & \underset{l=1}^{\mathrm{st.}} & \underset{l=r}{\sum} \sum_{k \in S_{1}^{F}} Q_{k}^{EH} \\ Q_{k}^{E} &= \sum_{l=1}^{F} \sum_{j \in P} \sum_{k \in S_{1}^{F}} Q_{k}^{EH} \\ & \underset{l=1}{\overset{m}{\sum}} \sum_{j \in P} \sum_{k \in S_{1}^{F}} \sum_{l=p^{k}} Y_{k}^{EU} = l \\ & \underset{l=1}{\overset{m}{\sum}} \sum_{j \in P} \sum_{k \in S_{1}^{F}} \sum_{i=p^{k}+1} Y_{k}^{EU} = l \\ & \underset{l=1}{\overset{m}{\sum}} \sum_{i \in P} \sum_{k \in S_{1}^{F}} \sum_{i=p^{k}+1, \dots, p^{k}} q_{k}^{EH} \\ & \underset{l=1}{\overset{m}{\sum}} \sum_{i=1} \sum_{j \in P} \sum_{i=1, p^{k}+1, \dots, p^{k}} q_{k}^{EH} \\ & \underset{l=1}{\overset{m}{\sum}} \sum_{i=1, p^{k}+1} Q_{k}^{EH} \\ & \underset{l=p^{k}+1}{\overset{m}{\sum}} Q_{k}^{EC} \\ & \underset{l=p^{k}+1}{\overset{m}{\sum}} Q_{k}^{EC} \\ & \underset{l=p^{k}+1}{\overset{m}{\sum}} Q_{k}^{EC} \\ & \underset{l=p^{k}+1}{\overset{m}{\sum}} Z_{k}^{E} | dX_{l} | \geq \sum_{i=p^{k}+1} q_{k}^{EH} \\ & \underset{l=p^{k}+1}{\overset{m}{\sum}} Z_{k}^{E} | dX_{l} | \geq \sum_{i=p^{k}+1} q_{k}^{EH} \\ & \underset{l=p^{k}+1}{\overset{m}{\sum}} Z_{k}^{E} | dX_{l} | \geq \sum_{i=p^{k}+1} q_{k}^{EH} \\ & \underset{l=p^{k}+1}{\overset{m}{\max}} \\ & \underset{l=p^{k}+1}{\overset{m}{\sum}} Z_{k}^{E} | dX_{l} | \geq \sum_{i=p^{k}} q_{k}^{EH} \\ & \underset{l=p^{k}+1}{\overset{m}{\max}} \\ & \underset{l=p^{k}+1}{\overset{m}{\max}} \\ & \underset{l=p^{k}+1}{\overset{m}{\sum}} \\ & \underset{l=p^{k}+1}{\overset{m}{\max}} \\ & \underset{l=p^{k}+1}{\overset{m}{\sum}} \\ & \underset{l=p^{k}+1}{\overset{m}{\max}} \\ & \underset{l=p^{k}+1}{\overset{$$

where l refers to the number of circuits that is increased until maximum savings are attained. To build this model, temperature constraints are added to model **P4** to guarantee that any circuit between two plants satisfies the second law. Moreover, the upper and lower temperatures for these cir-

cuits are represented by binary variables  $Y_{ikjl}^{EU}$  and  $Y_{ikjl}^{EL}$ , respectively. These variables allow heat transfer where the circuits span by setting variables  $Z_{ikjl}^{EU}$  to one; these variables are related to the interval heat transfer amounts by big M



Figure 10. Direct integration solution for Example 3.

constraints. All these constraints are direct extensions of the ones developed for the case of two plants. The reader is referred to Rodera and Bagajewicz (1999a) for a detailed explanation.

For simplicity, model **P6** considers unassisted heat integration only. Consideration of assisted cases requires the addition of the penalty term  $-\epsilon(Q_A + Q_B)$  in the objective function and the addition of similar constraints for circuits involving assisted heat for each point of the sequence. This is an MINLP model, and it becomes linear by using the Glover transformation (Glover, 1975), which consists of replacing the product of continuous variables times binary variables with a set of linear constraints (Rodera and Bagajewicz, 1999a).

The strategy applied to generate equal intervals for the case of targeting also has to be applied to establish the intervals used by model **P6**. However, this leads to the use of too many



Figure 11. Indirect integration solution for Example 3.

Table 8. Direct Integration for Example 3

		Savings (MW)	
Unit	Code	Heating	Cooling
Platformer (reformer)	PLAT	5.35	0.00
Visbreaker (thermal cracking)	VBU	0.46	0.00
Kerosene hydrotreater	KHT	0.64	0.90
Naphtha hydrotreater	NHT	3.86	1.60
Crude and vacuum distillation	CDU/VDU	9.16	11.84
Fluid-catalytic cracking	FCCU	0.00	3.53
Diesel hydrotreater	DHT	0.00	1.60
Total savings	—	19.47	19.47

 Table 9. Indirect Integration for Example 3

		Savings (MW)	
Unit	Code	Heating	Cooling
Platformer (reformer)	PLAT	6.03	0.00
Visbreaker (thermal cracking)	VBU	0.67	0.00
Kerosene hydrotreater	KHT	0.64	1.95
Naphtha hydrotreater	NHT	3.87	1.61
Crude and vacuum distillation	CDU/VDU	6.82	9.28
Fluid-catalytic cracking	FCCU	0.00	3.57
Diesel hydrotreater	DHT	0.00	1.62
Total savings	_	18.03	18.03

binary variables, thus making the MILP problem difficult to converge. The heat supplied and the heat demand decomposition are therefore proposed to alleviate the computational burden. For each plant, we propose to write a set of equations that establish the single circuits, making the equations independent of the interval partitioning. The equations are



Figure 12. Independent-circuits indirect integration solution for Example 2.

structed by using the starting temperatures of the individualplant streams, are partitioned further by considering the temperatures of the plants that will eventually deliver heat to, or receive heat from, the plant. These additional temperatures are shifted by adding or subtracting the minimum temperature difference from the original temperatures depending on

$$F_{kjl}^{E} \sum_{i=p^{k}+1}^{r} Z_{ikjl}^{EH} \Delta T_{i} \geq \sum_{i=p^{k}+1}^{r} q_{ikjl}^{EH} \quad \forall r = (p^{k}+1), \dots, (p^{j}-1) \\ F_{kjl}^{E} \sum_{i=p^{k}+1}^{p^{j}} Z_{ikjl}^{EH} \Delta T_{i} = \sum_{i=p^{k}+1}^{p^{j}} q_{ikjl}^{EH} \\ q_{ikjl}^{EH} - U_{ikjl}^{EH} Z_{ikjl}^{EH} Z_{ikjl} \leq 0 \qquad \forall i = (p^{k}+1), \dots, p^{j} \\ Z_{(p^{j}+1)kjl}^{EH} = Y_{(p^{j})kjl}^{EH} \\ Z_{ikjl}^{EH} = Z_{(i-1)kjl}^{EH} + Y_{(i-1)kjl}^{EH} - Y_{(i-1)kjl}^{EH} \qquad \forall i = (p^{k}+2), \dots, p^{j} \\ F_{jkl}^{E} \sum_{t=s}^{p^{k}} Z_{ijkl}^{EC} \Delta T_{t} \geq \sum_{t=s}^{p^{k}} q_{ijkl}^{EC} \\ F_{jkl}^{E} \sum_{t=s}^{p^{k}} Z_{ijkl}^{EC} \Delta T_{t} \geq \sum_{t=s}^{p^{k}} q_{ijkl}^{EC} \qquad \forall t = (p^{j}+1), \dots, p^{k} \\ q_{ijkl}^{E} - U_{ijkl}^{EC} Z_{ijkl}^{EC} \Delta T_{t} = \sum_{t=p^{j}+1}^{p^{k}} q_{ijkl}^{EC} \\ Z_{(p^{j}+1)kjl}^{EC} = Y_{(i-1)kjl}^{EC} + Y_{(i-1)kjl}^{EC} - Y_{(i-1)kjl}^{EC} \qquad \forall t = (p^{j}+2), \dots, p^{k} \\ Z_{(p^{j}+1)kjl}^{EC} = Z_{(i-1)kjl}^{EC} + Y_{(i-1)kjl}^{EC} - Y_{(i-1)kjl}^{EC} \qquad \forall t = (p^{j}+2), \dots, p^{k} \\ Z_{(kjl)}^{EC} = Z_{(i-1)kjl}^{EC} + Y_{(i-1)kjl}^{EC} - Y_{(i-1)kjl}^{EC} \qquad \forall t = (p^{j}+2), \dots, p^{k} \\ \end{cases}$$

Different binary variables and different variables that allow the heat transfer for the supplier and receiver sides are used. These two sets of equations are linked only by the circuit flow rates. Thus, the original intervals in each plant, con-

the direction of the heat transfer. The procedure is illustrated in Appendix B. The following equation is added to guarantee that the heat transfer from the circuit to the re-

		•		
		Heat Transfer	Total Heat	% Total
No. of		by Circuits	Transfer	Indirect
Circuits	Circuits	(kW)	(kW)	Savings
1	Ι	141.0	141.0	38.5
2	I,II	141.0, 107.5	248.5	67.8
3	I,II,III	141.0, 107.5, 84.9	333.4	90.9
4	I,II,III,IV	174.3, 107.5, 84.9, 33.3	366.7	100

 Table 10. Independent-Circuits Indirect Integration

 Solution for Example 2

ceiver is equal to the heat transfer from the supplier to the circuit.

$$\sum_{i=p^{k}+1}^{p^{j}} q_{ikjl}^{EH} = \sum_{t=p^{k}+1}^{p^{j}} q_{ikjl}^{EC} \quad k \in S_{j}^{E}; \quad \forall j \in P;$$
$$l = 1, \dots, N_{C}^{\min} \quad (12)$$

Finally, to connect both sets of constraints properly, extra relations are required to establish that the temperatures at the upper and lower parts of the circuits are the same in both the supplier and receiver plants. These constraints are

$$Y_{ikjl}^{EHU} = Y_{tkjl}^{ECU}$$

$$Y_{ikjl}^{EHL} = Y_{tkjl}^{ECL}$$

$$\forall (i,t) \in \{(i,t)/T_i = T_l\};$$

$$k \in S_i^E; \quad \forall j \in P; \quad l = 1, \dots, N_C^{\min} \quad (13)$$

If numerical problems are still a concern, further reduction of the number of intervals is possible. One can use the method of Appendix B to further partition the set of intervals that belong to the regions containing the connections shown in the solution of problem P5 and define binary variables for only these intervals. The resulting procedure is illustrated in Appendix C. In using this approach, one can only guarantee optimality when the solution states that all the targeted effective heat is transferred using the intervals proposed. When the solution states that less heat than the target is transferred, it might be possible that one circuit, positioned in other intervals, can transfer more. Thus, suboptimal solutions are possible.

It is important to note that the solutions to problem P6 are obtained under the assumption that all the plants performing the integration are in operation. As was analyzed in previous work (Bagajewicz and Rodera, 2000), when one or more plants are shut down, the heat integration between the others may

 Table 11. Independent-Circuits Indirect Integration

 Solution for Example 3

No. of		Heat Transfer	Total Heat	% Total
No. of Circuits	Circuits	(MW)	(MW)	Savings
1	I	5.12	5.12	28.4
2	I,II	5.12, 3.52	8.64	47.9
3	I,IÍ,III	3.86, 3.52, 3.85	11.23	62.3
4	I,II,III,IV	2.69, 3.52, 3.85, 2.67	12.73	70.6
5	I,II,III,IV,V	2.69, 3.52, 3.85, 2.67, 0.64	13.37	74.2

be affected. Because the problem is known to have alternative solutions, one could try to use those solutions that maximize the savings in circumstances for which different subsets of plants are not in operation. This results in a stochastic planning problem that is not addressed in this article.

# Example 2 (Continued)

Table 10 and Figure 12 show the results obtained after solving the sequence that locates an increasing number of independent circuits by solving problem **P6**. A minimum of three circuits that transfer effective heat are required to achieve maximum savings, one between plants 1 and 2, the second between plants 2 and 3, and the third between plants 3 and 4. An additional circuit that transfers assisting heat from plant 2 to plant 3 is also required to debottleneck the heat cascade of plant 2.

Notice that the number of effective and assisting connections is minimal, and it is obtained directly when solving problem **P6** for the location of the increasing number of independent circuits. The solution of problem **P5**, however, can be used *a priori* to reduce the number of intervals to be generated using the procedure presented in Appendix C.

# Example 3 (Continued)

The procedure that establishes an increasing number of independent intermediate-fluid circuits is applied now to the entire oil refinery example. Table 11 shows the results obtained by solving problem **P6** for the location of up to five independent intermediate fluid circuits. These five independent circuits are capable of transferring 74.2% of the maximum energy savings for the indirect integration of the entire oil refinery.

The circuits presented in Table 11 are shown in Figure 13. Notice that circuits I and IV are established between the same pair of plants (from plant 5 to plant 1). One could ask why these could not be merged in one circuit. Note first that circuit I transfers 5.12 MW when it is alone and/or when circuit II is added, but drops to 3.86 MW when circuit 3 is added.



Figure 13. Independent-circuits indirect integration solution for Example 3.

 Table 12. Steam Levels for Example 3

Steam Level	Pres. (psi)	Temp. (°C)	Heat Transfer (MW)
High pres. (HP)	600 250	254	5.01
Medium pres. (MP)	250	208	5.31

When circuit 4 is added, the total heat transfer between these two plants is 5.36 MW. If the circuits were merged into one, the transfer would be the equivalent of the 3.86 MW that was achieved when only circuit 3 was added. Thus, by establishing two circuits, which will carry different flow rates, one can achieve more savings.

Comparing Figure 13 with Figure 11, one can notice that plants 2 and 7 were left out. Plant 7 can deliver 2.45 MW, but this is delivered to four effective receivers. In addition, 0.82 MW of assisted heat are needed. Thus, one can anticipate that the number of circuits involved (five) is a large price to pay for such a small increment. The second plant left out from the solution shown in Figure 13 is plant 2. It receives 3.36 MW of effective heat from five effective suppliers (one of them being plant 7). It is also required to assist two plants with 2.69 MW. Thus, the number of circuits involved is likely to be six. Even if this number of circuits is reduced by exploring alternative solutions, one can perceive that what is left to integrate will require a small amount of heat savings per circuit added with a large amount of assisting circuits. Clearly, the incremental cost at this point is getting to be too large. In addition, one must note that circuit one keeps reducing the heat transferred as circuits are added, working against the gains achieved by adding new circuits.

# Use of Steam

It is common to find practitioners who claim that there is no need to resort to complicated intermediate fluid circuits. Rather, they postulate, the steam system suffices to attain sizable enough and/or profitable savings. The argument used is that the capital investment is minimum, because no piping is needed to transport process streams or intermediate fluids from one plant to the other. The issue was discussed by Rodera and Bagajewicz (1999a, 2001) showing that this is not always true. In the particular case of Example 3, two levels of steam are considered in Table 12 (high and medium pressure), revealing that only 10.32 MW of savings can be achieved. Figure 14 shows the total site profiles obtained by





the traditional method. Note that the savings that could be obtained with low-pressure steam are negligible. The filled lines represent the amounts of steam that are generated or consumed to produce the integration. The savings represent 57.2% of the total indirect savings compared with the 74.2% obtained by five intermediate fluid circuits, proving that aforementioned claims are not always true. At this point, one might want to consider how the utility system is affected by this solution. Since reduction of energy consumption is achieved, this interaction is the same as in the case of single plant heat recovery targeting and can be resolved in a second step.

#### Conclusion

A targeting method for heat integration between plants presented earlier by Rodera and Bagajewicz (1999a) was extended to consider a total site composed by a set of n plants. Important new aspects are revealed. The pattern corresponding to assisted heat transfer between two plants changes for many plants. In particular, assisting heat can be transferred in both opposite and parallel directions to the effective heat transfer. For indirect integration, transfer between equal intervals that are a fixed number of intervals apart is used to account for the presence of the fluid circuits. Finally, the resulting problem exhibits alternative solutions, and flexibility is gained by optimizing the different operational scenarios.

# Notation

- i = temperature interval
- j = chemical plant
- l = number of independent intermediate-fluid circuits between plant pairs
- k = auxiliary chemical plant
- m = total number of intervals
- n = total number of plants
- $p^{j}$  = last interval above the pinch of plant j
- $Q_A$  = total heat transferred in the zone above pinches
- $Q_B =$  total heat transferred in the zone below pinches
- $Q_E^-$  = total heat transferred in the zone of effective transfer of heat (between pinches)
- q = heat surplus or heat demand/heat transferred
- $q^{j}$  = heat surplus or heat demand in plant j
- $\hat{\delta}_0$  = minimum surplus to the first interval
- $\hat{\delta}_0 =$  original minimum surplus to the first interval
- $\delta$  = minimum cascaded heat
- $\hat{\delta}$  = original minimum cascaded heat

#### *Superscripts*

- A = zone above both pinches
- B = zone below both pinches
- E = zone of effective transfer of heat (between pinches)
- j = chemical plant

#### **Subscripts**

- A = zone above both pinches
- B = zone below both pinches
- E = zone of effective transfer of heat (between pinches)
- i = temperature interval
- j = chemical plant
- k = auxiliary chemical intervals

#### Sets

 $P = \text{set of } n \text{ plants considered for direct or indirect integration} R_j^A = \text{set of assisting plants } k \text{ receiving heat from plant } j \text{ in the region above the pinch}$ 

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- $S_i^A$  = set of assisted plants k supplying heat to plant j in the region above the pinch
- $R_i^E$  = set of effective-receiver plants k receiving heat from plant j
- $S_{i}^{E}$  = set of effective-supplier plants k supplying heat to plant j
- $R_{i}^{B}$  = set of assisted plants k receiving heat from plant j in the region below the pinch
- $S_i^B = \text{set of assisting plants } k$  supplying heat to plant j in the region below the pinch
- $R_{ij}^A$  = set of assisting plants  $k \in R_j^A$  present in interval *i*
- $S_{ii}^{A}$  = set of assisted plants  $k \in S_{i}^{A}$  present in interval i
- $R_{ii}^E$  = set of effective-receiver plants  $k \in R_i^E$  present in interval i
- $S_{ii}^{E}$  = set of effective-supplier plants  $k \in S_{i}^{E}$  present in interval i
- $R_{ij}^{B}$  = set of assisted plants  $k \in R_{j}^{B}$  present in interval *i*  $S_{ij}^{B}$  = set of assisting plants  $k \in S_{j}^{B}$  present in interval *i*

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# Appendix A

In this appendix, we present a procedure to obtain the generalized set of intervals used for indirect heat integration when solving models P4 to P7. In our previous article (Rodera and Bagajewicz, 1999a) the transfer of heat between plants was performed between intervals at the same temperature level. Such a thing can be accomplished by a special shifting of scales. This shifting cannot be performed when many plants are considered. Therefore, in this article the transfer between plants is modeled as an upward and downward diagonal transfer between equal size intervals that are a fixed number of intervals apart. We now present a procedure to obtain equal intervals.



Figure A1. Horizontal and diagonal heat transfer.

Consider the temperature intervals within the region between pinches for the case of two plants. A simple shift of the scales of plant 2 downward by  $\Delta T_{\min}$  degrees guarantees that the hot streams of plant 2 are at the same temperature of the cold streams of plant 1, and the use of an intermediate fluid is possible. For assisted cases, heat is transfer in the opposite direction to the effective heat transfer, and in the regions above and below both pinches. This requires two additional scale shifts and two gaps are generated (Rodera and Bagajewicz, 1999a). Figure A1a shows the final arrangement that allows horizontal heat transfer. To avoid conflicts for the case of more than two plants, this procedure is replaced in the region between pinches by diagonal transfer from an interval in plant 2 to an interval in plant 1 located  $\Delta T_{\min}$  degrees below. In the other regions, diagonal transfer results from an interval in plant 1 to an interval in plant 2 located  $\Delta T_{\min}$  degrees below. Figure A1b shows the diagonal transfer in all the regions. Note that corresponding intervals of equal size have to exist in both plants to make possible the heat transfer.

Starting from the uppermost temperature, new interval boundaries are generated by subtracting from each of the existing temperatures increasing number of fixed temperature differences ( $\Delta T_{\min}$ ). All the sequences are terminated at the closest temperature above the minimum existing temperature. The same thing is done starting from the lowermost temperature. In this last case, all the sequences are terminated at the closest temperature below the maximum existing temperature. The procedure is repeated for each original interval temperature boundary. Finally, all temperatures are sorted and the procedure ends. The reader can simply verify that the procedure guarantees equal size temperature intervals located  $\Delta T_{min}$  degrees one from the other in both upward and downward directions.

# Appendix B

In this appendix, we present a procedure to obtain the generalized set of intervals used for indirect heat integration when the equations for the circuits are decoupled, as shown in Eq. 11. Consider for simplicity that only effective heat transfer is present. A similar procedure is conducted when assisting heat integration is required. The original intervals in



Figure B1. Horizontal and diagonal heat transfer.

each plant constructed by using the starting temperatures of only streams belonging to each plant are shown in Figure B1a. The procedure consists of the further partition of the individual set of intervals in order to consider all the temperatures at which a circuit can start or end. From the point of view of a receiver plant, the circuit represents a hot stream delivering heat to its intervals. In addition to its original interval partition temperatures, a circuit can start or end at a supplier interval-partition temperature shifted  $\Delta T_{\rm min}$  degrees downward. This shift considers the fact that the circuit represents a cold stream for the supplier. Therefore, the temperature originally in the cold scale of the supplier will be now in the hot scale of the receiver. An equivalent analysis can be done from the point of view of the supplier where a circuit can start or end at an original interval partition or at a receiver interval partition temperature shifted  $\Delta T_{\rm min}$  degrees



Figure C1. Horizontal and diagonal heat transfer.

upward. Figure B1b shows the final interval partitions with the connecting dot lines representing possible starting/ending temperatures for the circuits.

This partitioning guarantees that all circuit temperatures will be included, but as Figure B1b shows, there is no direct relation between the interval number and the temperatures when a circuit is established. Therefore, Eq. 13 is added to relate temperatures in different plants. A simplification of the partition procedure presented above, which produces a large number of intervals but has simpler implementation, is to directly consider the intervals used for direct integration and further partition them by considering temperatures shifted  $\Delta T_{\rm min}$  degrees upward and  $\Delta T_{\rm min}$  degrees downward. The resulting number of intervals for the simplified case will be at most three times the number of intervals used when direct integration is considered.

# Appendix C

In this appendix, we present a procedure to obtain a set of intervals for indirect heat integration using the solution of model **P5**. Figure C1a shows an example of three plants where effective heat integration from plant 2 to plant 1 and from plant 3 to plant 2 is present. Also, assisting heat from plant 1 to plant 2 is required. With the use of this information, the procedure explained in Appendix B is applied only to these regions. That is, the original temperature intervals of each plant are further partitioned using temperatures of the receivers of suppliers only where heat transfer is predicted by model **P5**. The final interval partitions are shown in Figure C1b.

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