Alternative MILP Formulations for Shell and Tube Heat Exchanger Optimal Design

Caroline de O. Gonçalves,[‡] *André L. H. Costa,*[‡] *Miguel J. Bagajewicz**[†]

[‡]Institute of Chemistry, Rio de Janeiro State University (UERJ), Rua São Francisco Xavier, 524, Maracanã, Rio de Janeiro, RJ, CEP 20550-900, Brazil

[†]School of Chemical, Biological and Materials Engineering, University of Oklahoma, Norman Oklahoma 73019

ABSTRACT

In a recent article (Gonçalves et al., 2016), we presented an MILP formulation for the detailed design of heat exchangers. The formulation relies on the use of standardized values for several mechanical parts expressed in terms of discrete choices applied to one simple model (Kern, 1950). Because we consider that this model could be used as part of more complex models (i.e. HEN synthesis), in this article we explore several different modeling options to speed-up computational time. These options are based on different alternatives of aggregation of the discrete values in relation to the set of binary variables. Numerical results show that these procedures allow large computational effort reductions.

1. INTRODUCTION

Heat exchangers are equipment responsible for the modification of the temperature and/or physical state of process streams. They are a considerable fraction of the hardware of process industries, where a large process plant (e.g. a refinery) may involve the design of several hundreds of heat exchangers (Buzek and Podkanski, 1996).

The traditional approach for the design of heat exchangers involves the direct intervention of a skilled engineer in a trial-and-error procedure. Most often, the main target is the identification of a feasible heat exchanger candidate able to fulfill the desired thermal service. Since, for a given thermal task, there are different feasible alternatives, the quality of the design is highly dependent on the experience of the engineer. This aspect becomes even more important in a scenario of generational transition, where engineering teams were reduced and thermal specialists became rare in chemical and oil companies (Butterworth, 2004). Modern textbooks present algorithms for the solution of the design problem, where some level of optimization is included, but these schemes keep the same trial-and-error logic (Cao, 2009).

Aiming at circumventing the limitations of the traditional design approach, several papers formulate the design problem as an optimization problem (Caputo et al., 2015). The objective function is usually the minimization of the heat exchanger area restricted by allowable pressure drops or the minimization of the total annualized cost, including capital and operating costs in a yearly basis (Jegede and Polley, 1992). The main constraints are the thermal and hydraulic equations of the heat exchanger model.

In general, the computational techniques employed for the solution of the design problem can be classified into three categories: heuristic, metaheuristic and mathematical programming. The heuristic methods explore the search space based on thermo-fluid dynamic relations with the support of graphics (Muralikrishna and Shenoy, 2000) or screening tools (Ravagnani et al., 2003). Metaheuristic methods consists of randomized algorithms for the search of the optimal solution, such as, simulated annealing (Chaudhuri and Diwekar, 1999), genetic algorithms (Ponce-Ortega et al., 2009), particle swarm optimization (Sadeghzadeh et al., 2015), among others. Our article is inserted into the third category: mathematical programing. Mathematical programming techniques involve the utilization of deterministic algorithms, where the solution can be found based on formal optimality conditions (local or global). Newer mathematical programming solutions for the design of heat exchangers consider the discrete nature of the design variables, thus yielding mixed-integer nonlinear programming (MINLP) problems (Mizutani et al., 2003; Ponce-Ortega et al., 2006; Ravagnani and Caballero, 2007). An important aspect of MINLP alternatives is their nonconvexity, which may present nonconvergence problems and multiple local optima.

Recently, we proposed a mixed-integer linear programming (MILP) formulation for the design problem (Gonçalves et al., 2016), aiming the minimization of the heat transfer area. The model is based on the utilization of standard values for several mechanical parts expressed in terms of discrete choices together with one simple hydraulic and thermal model (Kern, 1950). For example, tube diameters come only in certain discrete values of diameter and their wall thickness dictated by a BWG scale. The same goes for shell diameters, tube length, etc.

The model presented by Gonçalves et al. (2016) makes use of several binary variables, representing the discrete options of the geometric parameters. When using these discrete representations together with the nonlinear equations corresponding to the calculation of heat transfer coefficients (shell, tube and overall), and the pressure drop on both tube and shell sides, the resulting model is a MINLP. We attempted to solve this MINLP model and obtained local minima in several cases. However, when the discrete variables are substituted and several algebraic conversions are made, the resulting model is rigorously linear. Work is underway to apply this methodology to other more modern hydraulic and thermal models (e.g. Bell-Delaware and stream analysis).

Despite the MILP superiority in relation to the reduction of the objective function and convergence when compared to the MINLP version, computational times employed are high. Because we aim at this model to be used as part of more complex models (i.e. HEN synthesis), and there is a need to improve computational efficiency, the focus of this paper is to present alternative MILP formulations aiming to reduce the computational effort.

For each standard value of a design variable, Gonçalves et al (2016) used a corresponding set of binary variables in their MILP model. The same direct relation between design and binary variables was also employed in Mizutani et al. (2003). There are, however, some alternatives in the literature. Ravagnani and Caballero (2007) used heat exchanger counting tables to describe some of the discrete values, where each combination of geometric parameters, corresponding to a counting table row, is associated to a single binary variable.

This paper investigates different aggregation options of the discrete values and the corresponding binary variables, to improve the computational performance of the MILP solution algorithm.

The article is organized as follows: For completion, we first present the non-linear MINLP model as presented by Gonçalves et al. (2016), which it is used as starting point to the MILP formulation development. We then discuss the alternative discrete representations and use one option to present the resulting linear model, which is similar, but not equal to the model presented in the previous article. We then discuss the computational performance results obtained using different options.

2. HEAT EXCHANGER MODEL

2.1. Scope. Our optimization problem corresponds to the design of shell and tube heat exchangers with a single E-type shell with single segmental baffles, applied for services without phase change in turbulent flow. There are seven design variables: number of tube passes (*Ntp*), tube diameter (outer and inner: *dte* and *dti*), tube layout (*lay*), tube pitch ratio (*rp*), number of baffles (*Nb*), shell diameter (*Ds*) and tube length (*L*). The fluid allocation is assumed previously established by the designer and is not included in the optimization.

The next subsections present the nonlinear model of the heat exchanger design problem that is employed as starting point for the development of all linear formulations compared in this paper. Here, the fixed parameters established prior the optimization are represented with the symbol "^".

2.2. Shell-Side Thermal and Hydraulic Equations. The convective heat transfer coefficient is evaluated using the Kern model (Kern, 1950), relating Nusselt (*Nus*), Reynolds (*Res*), and Prandtl numbers ($\widehat{Prs}$):

$$Nus = 0.36\, Res^{0.55} \widehat{Prs}^{1/3} \tag{1}$$

$$Nus = \frac{hs\, Deq}{\widehat{ks}} \tag{2}$$

$$Res = \frac{Deq\, vs\, \widehat{\rho s}}{\widehat{\mu s}} \tag{3}$$

$$\widehat{Prs} = \frac{\widehat{Cps}\, \widehat{\mu s}}{\widehat{ks}} \tag{4}$$

where *hs* is the shell-side convective heat transfer coefficient, *vs* is the flow velocity, and *Deq* is the equivalent diameter. The thermophysical properties are specific mass, $\widehat{\rho s}$, heat capacity, $\widehat{Cps}$, dynamic viscosity, $\widehat{\mu s}$, and thermal conductivity, $\widehat{ks}$.

The evaluation of the equivalent diameter depends on the tube layout:

$$Deq = \frac{4\, ltp^2}{\pi\, dte} - dte \quad \text{(Square pattern)} \tag{5}$$

$$Deq = \frac{3.46\, ltp^2}{\pi\, dte} - \; dte \quad \text{(Triangular pattern)} \tag{6}$$

where *ltp* is the tube pitch.

The expression of the shell-side flow velocity is:

$$vs = \frac{\widehat{ms}}{\widehat{\rho s}\, Ar} \tag{7}$$

where $\widehat{ms}$ is the mass flow rate. The flow area in the shell-side flow is given by:

$$Ar = \; Ds\; FAR\; lbc \tag{8}$$

where *lbc* is the baffle spacing. The expression of the free-area ratio, *FAR*, is:

$$FAR = \frac{(ltp - dte)}{ltp} = 1 - \frac{1}{rp} \tag{9}$$

The head loss in the shell-side flow is also based on the Kern model (Kern, 1950):

$$\frac{\Delta Ps}{\widehat{\rho s}\; \hat{g}} = \; fs \frac{Ds(Nb + 1)}{Deq} \left(\frac{vs^2}{2\; \hat{g}} \right) \tag{10}$$

where ΔPs is the shell-side pressure drop, and *fs* is the shell-side friction factor.

The shell-side friction factor is given by:

$$fs = \; 1.728\; Res^{-0.188} \tag{11}$$

The relation between the number of baffles and the baffle spacing is:

$$Nb = \frac{L}{lbc} - \; 1 \tag{12}$$

2.3. Tube-Side Thermal and Hydraulic Equations. The convective heat transfer coefficient is evaluated using the Dittus-Boelter correlation (Incropera and DeWitt, 2006), relating Nusselt (*Nut*), Reynolds (*Ret*) and Prandtl numbers ($\widehat{Prt}$) of the tube-side flow:

$$Nut = \; 0.023\; Ret^{0.8} \widehat{Prt}^{n} \tag{13}$$

$$Nut = \frac{ht\; dti}{\widehat{kt}} \tag{14}$$

$$Ret = \frac{dti\; vt\; \widehat{\rho t}}{\widehat{\mu t}} \tag{15}$$

$$\widehat{Prt} = \frac{\widehat{Cpt}\ \widehat{\mu t}}{\widehat{kt}} \qquad (16)$$

where *ht* is the convective heat transfer coefficient, *vt* is the flow velocity, $\widehat{\rho t}$ is the specific mass, $\widehat{Cpt}$ is the heat capacity, $\widehat{\mu t}$ is the dynamic viscosity, $\widehat{kt}$ is the thermal conductivity, and the parameter *n* is equal to 0.4 for heating and 0.3 for cooling.

The expression of the flow velocity in the tube-side is:

$$vt = \frac{4\ \widehat{mt}}{Ntp\ \pi\ \widehat{\rho t}\ dti^2} \qquad (17)$$

where $\widehat{mt}$ is the mass flow rate and density, and *Ntp* is the number of tubes per pass.

The pressure drop in the tube-side flow is given by (Saunders, 1988):

$$\frac{\Delta Pt}{\widehat{\rho t}\ \hat{g}} = \frac{ft\ \ Npt\ \ L\ \ vt^2}{2\ \hat{g}\ dti} + \frac{K\ \ Npt\ \ vt^2}{2\ \hat{g}} \qquad (18)$$

where *ft* is the tube-side friction factor. The parameter *K*, associated to the pressure drop in the heads, is equal to 0.9 for one tube pass and 1.6 for two or more tube passes.

The Darcy friction factor for turbulent flow is given by (Saunders, 1988):

$$ft = 0.014 + \frac{1.056}{Ret^{0.42}} \qquad (19)$$

2.4. Heat Transfer Rate Equation and Overall Heat Transfer Coefficient.

Based on the LMTD method, and considering a design margin ("excess area", $\widehat{Aexc}$), the heat transfer area must obey the following relation:

$$UA \geq \left(1 + \frac{\widehat{Aexc}}{100}\right)\frac{\hat{Q}}{\widehat{\Delta Tlm}\ F} \qquad (20)$$

where *U* is the overall heat transfer coefficient, *A* is the heat transfer area, $\hat{Q}$ is the heat load, $\widehat{\Delta Tlm}$ is logarithmic mean temperature difference (LMTD), and *F* is the LMTD correction factor (Incropera and DeWitt, 2006).

The area of the heat exchanger (*A*) depends on the total number of tubes (*Ntt*):

$$A = Ntt\ \ \pi\ dte\ L \qquad (21)$$

The expression for the evaluation of the overall heat transfer coefficient (U) is:

$$U = \frac{1}{\frac{dte}{dti\,ht} + \frac{\widehat{Rft}\,dte}{dti} + \frac{dte \ln(\frac{dte}{dti})}{2\,\widehat{ktube}} + \widehat{Rfs} + \frac{1}{hs}} \tag{22}$$

where $\widehat{ktube}$ is the thermal conductivity of the tube wall, and $\widehat{Rft}$ and $\widehat{Rfs}$ are the tube-side and shell-side fouling factors.

The LMTD correction factor is equal to 1, for one tube pass and is equal to the following expression for an even number of tube passes:

$$\hat{F} = \frac{(\hat{R}^2 + 1)^{0.5} \ln\left(\frac{(1-\hat{P})}{(1-\hat{R}\,\hat{P})}\right)}{(\hat{R}-1)\ln\left(\frac{2-\hat{P}\left(\hat{R}+1-(\hat{R}^2+1)^{0.5}\right)}{2-\hat{P}\left(\hat{R}+1+(\hat{R}^2+1)^{0.5}\right)}\right)} \tag{23}$$

where:

$$\hat{R} = \frac{\widehat{Thi} - \widehat{Tho}}{\widehat{Tco} - \widehat{Tci}} \tag{24}$$

$$\hat{P} = \frac{\widehat{Tco} - \widehat{Tci}}{\widehat{Thi} - \widehat{Tci}} \tag{25}$$

2.5. Bounds on Pressure Drops, Flow Velocities and Reynolds Numbers.

The lower and upper bounds on pressure drops, velocities and Reynolds numbers are represented by:

$$\Delta Ps \leq \widehat{\Delta Psdisp} \tag{26}$$

$$\Delta Pt \leq \widehat{\Delta Ptdisp} \tag{27}$$

$$vs \geq \widehat{vsmin} \tag{28}$$

$$vs \leq \widehat{vsmax} \tag{29}$$

$$vt \geq \widehat{vtmin} \tag{30}$$

$$vt \leq \widehat{vtmax} \tag{31}$$

$$Res \geq 2 \cdot 10^3 \tag{32}$$

$$Ret \geq 10^4 \tag{33}$$

2.6. Geometric Constraints. Design recommendations and TEMA standards impose the following set of constraints (Taborek, 2008a):

$$lbc \geq 0.2\,Ds \tag{34}$$

$$lbc \leq 1.0\,Ds \tag{35}$$

$$L \geq 3\,Ds \tag{36}$$

$$L \leq 15\,Ds \tag{37}$$

2.7. Objective Function. The objective function of the optimization is the minimization of the heat transfer area:

$$\min \quad A \tag{38}$$

2.8. Discrete Variables As anticipated above, several variables can only adopt discrete values according to engineering practice (Taborek, 2008a,b,c) and TEMA standards (TEMA, 2007). They are: inner and outer tube diameter (*dti* and *dte*), tube length (*L*), number of baffles (*Nb*), number of tube passes (*Npt*), pitch ratio (*rp*), shell diameter (*Ds*), and tube layout (*lay*). Thus, we substitute the following expressions in the above presented model.

$$x = \sum_i \widehat{xd}_i\, y_i \tag{39}$$

$$\sum_i y_i = 1 \tag{40}$$

where *x* represents a generic discrete variable, $\widehat{xd}_i$ the value of option *I* for this variable and y_i a binary variable that is used to make the model choose one and only one option.

2.9. MILP Model. After the substitution of the discrete variables is made, the model results in a complex mixed integer nonlinear programming (MINLP) model that contains products of binaries and continuous variables. In our previous contribution (Gonçalves et al., 2016), we converted this rigorous MINLP model into a rigorous linear (MILP) model, making no simplifying assumptions. Thus, a rigorous solution of the MILP is also a rigorous solution of the MILP. Moreover, because of linearity, the MILP model renders a global solution. As we shown in our previous paper, solving the MINLP model using local solvers many times rendered a local solution that is not global.

2.10. MILP Model Performance. Once several options of binary variable prioritization in the MILP branch and bound, we came up with one option that rendered solutions in the range from 171 to 2824 seconds, with an average of 1458 seconds for 10 test problems. While this performance time is more than acceptable for a stand-alone run, even if the number of geometric options is increased. However, this computational time is still high when for example, repeated runs are needed to handle uncertainty, and when the model becomes a sub-model of others, like the simultaneous design of a heat exchanger network with detailed heat exchanger design. We now explore different rigorous alternatives of binary variable aggregation, all having different computational efficiency still rendering the same result.

3. ALTERNATIVES OF BINARY VARIABLES ORGANIZATION

We present five different aggregation of binary variables leading to MILP formulations, that render the same result each with its own computational efficiency.

3.1. Alternative 1. In this alternative, each set of binary variables corresponds to a discrete variable referred to as seen in the work of Gonçalves et al. (2016). Therefore, yd_{sd} corresponds to variable representing the tube diameter, yDs_{sDs} corresponds to shell diameter, yL_{sL} corresponds to tube length, $ylay_{slay}$ corresponds to tube layout, yNb_{sNb} corresponds to number of baffles, $yNpt_{sNpt}$ corresponds to number of tube passes, and yrp_{srp} corresponds to tube pitch ratio.

3.2. Alternative 2. A counting table structure can be employed to organize the discrete values of the shell diameter, tube diameter, tube layout, number of tube passes, and tube pitch ratio, where only one set of binary variables, $yrow_{srow}$, is employed to represent these discrete values. In this context, *srow* is a multi-index set, i.e. *srow* = (*sd*, *sDs*, *slay*, *sNpt*, *srp*). The tube length and the number of baffles remain represented by the original sets of binary variables yL_{sL} and yNb_{sNb}.

3.3. Alternative 3. This alternative represents the discrete values in two tables. The first one corresponds to the counting table, as shown in the previous alternative, where the corresponding set of binaries is $yrow1_{srow1}$ with *srow*1 = (*sd, sDs, slay, sNpt, srp*). The second table contains all pairs of discrete values of tube length and number of baffles. The set of binaries which represent these discrete values is $yrow2_{srow2}$ with *srow*2 = (*sNb sL*).

3.4. Alternative 4. Another possible combination was the use of two set of binary variables: $yrow1_{srow1}$ with *srow*1 = (*sd, sDs, slay, sNpt, srp, sL*), representing all variables but the number of baffles, which is represented by the original binary yNb_{sNb}.

3.5. Alternative 5. The last alternative investigated in this work is the use of a unique set of binary variables, $yrow_{srow}$, which corresponds to all discrete variables, $srow$ = (sd, sDs, $slay$, $sNpt$, srp, sL, sNb).

Table 1 contains an overview of the different combinations between binary variables and the original discrete variables.

Table 1. Alternatives investigated of binary variables

alternative	binary variable {original discrete variable}
1	yd_{sd} $\{dt\}$, yDs_{sDs} $\{Ds\}$, $ylay_{slay}$ $\{lay\}$, $yNpt_{sNpt}$ $\{Npt\}$ yrp_{srp} $\{rp\}$, yL_{sL} $\{L\}$, $yNb_{sNb}\{Nb\}$
2	$yrow_{srow}$ $\{dt, Ds, lay, Npt, rp\}$, yL_{sL} $\{L\}$, yNb_{sNb} $\{Nb\}$
3	$yrow_{srow1}$ $\{dt, Ds, lay, Npt, rp\}$, $yrow_{srow2}$ $\{L, Nb\}$
4	$yrow_{srow}$ $\{dt, Ds, lay, Npt, rp, L\}$, $yNb_{sNb}\{Nb\}$
5	$yrow_{srow}$ $\{dt, Ds, lay, Npt, rp, L, Nb\}$

4. DEVELOPMENT OF THE MILP FORMULATIONS

The new MILP formulations are built starting from the MINLP model (eqs. 1-38) through three main steps: the organization of the data table of the discrete variables, the model reformulation, and the conversion to a linear model. We outlined above the linearization procedure of Alternative 1, referring the reader to our previous article (Gonçalves et al., 2016). For reasons of space and because the procedure is very similar when aggregates of binary variables is made, we only illustrate Alternative 5 in detail (this alternative is associated to the highest reduction of the computational time consumed by the MILP solver, as it will be shown in the results). The mathematical formulations of the other alternatives are available in the Supporting Information.

4.1. Organization of the Data Table of the Discrete Variables. For Alternative 1, we have the following equations

$$dte = \sum_{sd=1}^{sdmax} \widehat{pdte}_{sd}\, yd_{sd} \tag{41}$$

$$dti = \sum_{sd=1}^{sdmax} \widehat{pdti}_{sd}\, yd_{sd} \tag{42}$$

$$Ds = \sum_{sDs=1}^{sDsmax} \widehat{pDs}_{sDs}\, yDs_{sDs} \tag{43}$$

$$lay = \sum_{slay=1}^{slaymax} \widehat{play}_{slay}\, ylay_{slay} \tag{44}$$

$$Npt = \sum_{sNpt=1}^{sNptmax} \widehat{pNpt}_{sNpt}\, yNpt_{sNpt} \tag{45}$$

$$rp = \sum_{srp=1}^{srpmax} \widehat{prp}_{srp} yrp_{srp} \tag{46}$$

$$L = \sum_{sL=1}^{sLmax} \widehat{pL}_{sL}\, yL_{sL} \tag{47}$$

$$Nb = \sum_{sNb=1}^{sNbmax} \widehat{pNb}_{sNb}\, yNb_{sNb} \tag{48}$$

with the following equations needed to guarantee only one choice among many:

$$\sum_{sd=1}^{sdmax} yd_{sd} = 1 \tag{49}$$

$$\sum_{sDs=1}^{sDsmax} yDs_{sDs} = 1 \tag{50}$$

$$\sum_{slay=1}^{slaymax} ylay_{slay} = 1 \tag{51}$$

$$\sum_{sNpt=1}^{sNptmax} yNpt_{sNpt} = 1 \tag{52}$$

$$\sum_{srp=1}^{srpmax} yrp_{srp} = 1 \tag{53}$$

$$\sum_{sL=1}^{sLmax} yL_{sL} = 1 \tag{54}$$

$$\sum_{sNb=1}^{sNbmax} yNb_{sNb} = 1 \tag{55}$$

According to the aggregation strategy employed in the development of the new MILP formulations, the parameters that represent the discrete values can be grouped in one or more tables. Therefore, several discrete values of the design variables are identified by the same index (a multi-index related to the corresponding original indices). For example, in Alternative 5, the multi-index *srow* represents the discrete values of all design variables. The corresponding set of parameters which compose the table are defined from the original ones, as follows

$$\widehat{Pdte}_{srow} = \widehat{pdte}_{sd} \tag{56}$$

$$\widehat{Pdti}_{srow} = \widehat{pdti}_{sd} \tag{57}$$

$$\widehat{PDs}_{srow} = \widehat{pDs}_{sDs} \tag{58}$$

$$\widehat{Play}_{srow} = \widehat{play}_{slay} \tag{59}$$

$$\widehat{PNpt}_{srow} = \widehat{pNpt}_{sNpt} \tag{60}$$

$$\widehat{Prp}_{srsrow} = \widehat{prp}_{srp} \tag{61}$$

$$\widehat{PL}_{srow} = \widehat{pL}_{sL} \tag{62}$$

$$\widehat{PNb}_{srow} = \widehat{pNb}_{sNb} \tag{63}$$

Consequently, different discrete variables become associated to the same set of binaries. In Alternative 5, all discrete variables are described by the set of binaries $yrow_{srow}$, thus yielding:

$$dte = \sum_{srow} \widehat{Pdte}_{srow}\, yrow_{srow} \tag{64}$$

$$dti = \sum_{srow} \widehat{Pdti}_{srow}\, yrow_{srow} \tag{65}$$

$$Ds = \sum_{srow} \widehat{PDs}_{srow}\, yrow_{srow} \tag{66}$$

$$lay = \sum_{srow} \widehat{Play}_{srow}\, yrow_{srow} \tag{67}$$

$$Npt = \sum_{srow} \widehat{PNpt}_{srow}\, yrow_{srow} \tag{68}$$

$$rp = \sum_{srow} \widehat{Prp}_{srow} yrow_{srow} \tag{69}$$

$$L = \sum_{srow} \widehat{PL}_{srow}\, yrow_{srow} \tag{70}$$

$$Nb = \sum_{srow} \widehat{PNb}_{srow}\, yrow_{srow} \tag{71}$$

$$\sum_{srow} yrow_{srow} = 1 \tag{72}$$

4.2. Model Reformulation. In this step, the model equations are modified through the substitution of the discrete variables by their binary representation. This reformulation step also involves a procedure for the organization of the resultant expressions containing binary variables, as described in the following paragraphs.

As stated, the relation between a design variable x and their discrete values $\widehat{xd}_i$, using binary variables y_i, is expressed by eqs. (39) and (40), where I can be a multi-index.

The substitution of a set of discrete variables p, q, … , z by its binary representation in the heat exchanger model yields terms of the form $p^{n1} q^{n2} \cdots z^{nm}$ that are substituted as follows:

$$p^{n1} q^{n2} \cdots z^{nm} = [\sum_i \widehat{pd}_i\, yp_i]^{n1}\, [\sum_j \widehat{qd}_j\, yq_j]^{n2}\, [\sum_k \widehat{zd}_k\, yz_k]^{nm} \tag{73}$$

Because all binary variables are equal to 1 only once in the corresponding set, this equation is equivalent to:

$$p^{n1} q^{n2} \cdots z^{nm} = \sum_{i,j,..k} \widehat{pd}_i^{\,n1} \widehat{qd}_j^{\,n2} \ldots\ldots \widehat{qd}_k^{\,nm}\; yp_i\; yq_j \ldots yz_k \tag{74}$$

After the application of this procedure, the reformulated model becomes composed of several expressions containing multiple summations of products of binary variables.

4.3. Conversion to a Linear Model. The product of binaries obtained from the discrete variable substitution can be reorganized in equivalent linear expressions, as discussed below.

Let the product of binaries be substituted by a variable $w_{i,j,\ldots,k}$:

$$p^{n1} q^{n2} \cdots z^{nm} = \sum_{i,j,..k} \widehat{pd}_i^{\,n1} \widehat{qd}_j^{\,n2} \ldots\ldots \widehat{qd}_k^{\,nm}\; w_{i,j,\ldots k} \tag{75}$$

where:

$$w_{i,j,\dots,k} = yp_i \, yq_j \dots yz_k \tag{76}$$

However, the nonlinearity existent in this equation can be eliminated through the substitution of this expression by the equivalent set of linear inequality constraints:

$$w_{i,j,\dots,k} \le yp_i \tag{77}$$

$$w_{i,j,\dots,k} \le yq_j \tag{78}$$

...

$$w_{i,j,\dots,k} \le yz_k \tag{79}$$

$$w_{i,j,\dots,k} \ge yp_i + yq_j + \dots + yz_k - (m-1) \tag{80}$$

where *m* is the number of binary variables in the product. Since Alternative 5 contains only one set of binary variables, this step is not necessary in its development, but it is fundamental to the other alternatives with lower aggregation levels.

5. MILP FORMULATION WITH A SINGLE SET OF BINARIES

This section presents the complete linear formulation of the optimal heat exchanger design problem based on a unique set of binary variables to represent the discrete options of the design variables (Alternative 5).

5.1. Binary Variables Equality Constraints. This constraint imposes that only one design alternative must be chosen:

$$\sum_{srow} yrow_{srow} = 1 \tag{81}$$

5.2. Heat Transfer Rate Equation. The expressions of all heat transfer coefficients and the heat transfer area are inserted into the heat transfer equation, thus yielding:

$$\hat{Q}\left(\sum_{srow}\frac{\widehat{Pdte}_{srow}}{\widehat{Pht}_{srow}\,\widehat{Pdti}_{srow}}yrow_{srow}+\ \widehat{Rft}\ \sum_{srow}\frac{\widehat{Pdte}_{srow}}{\widehat{Pdti}_{srow}}yrow_{srow}+\right.$$

$$\left.\frac{\sum_{srow}\widehat{Pdte}_{srow}\,yrow_{srow}\ln\left(\frac{\widehat{Pdte}_{srow}}{\widehat{Pdti}_{srow}}\right)}{2\ \widehat{ktube}}+\ \widehat{Rfs}+\sum_{srow}\frac{1}{\widehat{Phs}_{srow}}yrow_{srow}\right)\leq$$

$$\left(\frac{100}{100+\widehat{Aexc}}\right)\left(\pi\ \sum_{srow}\widehat{PNtt}_{srow}\,\widehat{Pdte}_{srow}\,\widehat{PL}_{srow}yrow_{srow}\right)\widehat{\Delta Tlm}\ \hat{F}_{srow} \tag{82}$$

where $\widehat{PNtt}_{srow}$is the total number of tubes and:

$$\widehat{Pht}_{srow}=\frac{\widehat{kt}\ 0{,}023\left(\frac{4\ \widehat{mt}}{\pi\ \widehat{\mu t}}\right)^{0{,}8}\widehat{Prt}^{n}}{\widehat{Pdti}_{srow}^{1{,}8}}\left(\frac{\widehat{PNpt}_{srow}}{\widehat{PNtt}_{srow}}\right)^{0{,}8} \tag{83}$$

$$\widehat{Phs}_{srow}=\frac{\widehat{ks}\ 0{,}36\left(\frac{\widehat{ms}}{\widehat{\mu s}}\right)^{0{,}55}\widehat{Prs}^{1/3}}{\widehat{PDeq}_{srow}{}^{0{,}45}}\left(\frac{(\widehat{PNb}_{srow}+1)}{\widehat{PDs}_{srow}\ \widehat{PFAR}_{srow}\ \widehat{PL}_{srow}}\right)^{0{,}55} \tag{84}$$

$$\widehat{PFAR}_{srow}=1-\frac{1}{\widehat{Prp}_{srow}} \tag{85}$$

$$\widehat{pDeq}_{srow}=\frac{\widehat{aDeq}_{srow}\widehat{Prp}_{srow}^{2}\widehat{Pdte}_{srow}^{2}}{\pi\widehat{Pdte}_{srow}}-\widehat{Pdte}_{srow} \tag{86}$$

$$\widehat{aDeq}_{srow}=\begin{cases}4 & \text{if } splay=1\\ 3.46 & \text{if } slay=2\end{cases} \tag{87}$$

$$\hat{F}_{srow}=\begin{cases}\dfrac{(\hat{R}^2+1)^{0{,}5}\ \ln\left(\frac{(1-\hat{P})}{(1-\hat{R}\,\hat{P})}\right)}{(\hat{R}-1)\ln\left(\frac{2-\hat{P}\left(\hat{R}+1-(\hat{R}^2+1)^{0{,}5}\right)}{2-\hat{P}\left(\hat{R}+1+(\hat{R}^2+1)^{0{,}5}\right)}\right)} & \text{if } sNpt\neq 1\\ 1 & \text{if } sNpt=1\end{cases} \tag{88}$$

5.3. Bounds on Pressure Drops, Flow Velocities and Reynolds Numbers.

The bounds on the shell-side and tube-side pressure drops are expressed by:

$$\sum_{srow}\widehat{P\Delta Ps}_{srow}yrow_{srow}\ \leq\ \widehat{\Delta Psdisp} \tag{89}$$

$$\sum_{srow}P\widehat{\Delta Ptturb}1_{srow}yrow_{srow}+\sum_{srow}P\widehat{\Delta Ptturb}2_{srow}yrow_{srow}+$$

$$\sum_{srow}P\widehat{\Delta Ptcab}_{srow}\hat{K}_{srow}yrow_{srow}\leq\widehat{\Delta Ptdisp} \tag{90}$$

where:

$$\widehat{P\Delta Ps}_{srow} = 0.864 \frac{\widehat{ms}^{1,812}\widehat{\mu s}^{0,188}}{\widehat{\rho s}} \left(\frac{(\widehat{PNb}_{srow}+1)^{2,812}}{\widehat{PDs}_{srow}{}^{0,812}(\widehat{PFAR}_{srow}\widehat{PL}_{srow})^{1,812}(\widehat{PDeq}_{srow})^{1,188}} \right) \tag{91}$$

$$P\Delta\widehat{Ptturb1}_{srow} = \left(\frac{0,112\ \widehat{mt}^2}{\pi^2\rho t}\right)\left(\frac{\widehat{PNpt}_{srow}^3\widehat{PL}_{srow}}{\widehat{PNtt}_{srow}^2\widehat{Pdti}_{srow}{}^5}\right) \tag{92}$$

$$P\Delta\widehat{Ptturb2}_{srow} = (0,528)\left(\frac{4^{1,58}\ \widehat{mt}^{1,58}\ \widehat{\mu t}^{0,42}}{\pi^{1,58}\ \widehat{\rho t}}\right)\frac{\widehat{PNpt}_{srow}^{2,58}\widehat{PL}_{srow}}{\widehat{pNtt}_{srow}^{1,58}\widehat{Pdti}_{srow}^{4,58}} \tag{93}$$

$$P\Delta\widehat{Ptcab}_{srow} = \left(\frac{8\ \widehat{mt}^2}{\pi^2\ \widehat{\rho t}}\right)\frac{\widehat{PNpt}_{srow}^3}{\widehat{PNtt}_{srow}^2\widehat{Pdti}_{srow}{}^4} \tag{94}$$

The bounds on the shell-side and tube-side flow velocities are:

$$\widehat{vsmin} \leq \frac{\widehat{ms}}{\widehat{\rho s}}\sum_{srow}\frac{(\widehat{PNb}_{srow}+1)}{\widehat{PDs}_{srow}\ \widehat{PFAR}_{srow}\ \widehat{PL}_{srow}}yrow_{srow} \tag{95}$$

$$\widehat{vsmax} \geq \frac{\widehat{ms}}{\widehat{\rho s}}\sum_{srow}\frac{(\widehat{pNb}_{srow}+1)}{\widehat{pDs}_{srow}\ \widehat{PFAR}_{srow}\ \widehat{PL}_{srow}}yrow_{srow} \tag{96}$$

$$\widehat{vtmin} \leq \frac{4\ \widehat{mt}}{\pi\ \widehat{\rho t}}\sum_{srow}\frac{\widehat{PNpt}_{srow}}{\widehat{PNtt}_{srow}\ \widehat{Pdti}_{srow}{}^2}yrow_{srow} \tag{97}$$

$$\widehat{vtmax} \geq \frac{4\ \widehat{mt}}{\pi\ \widehat{\rho t}}\sum_{srow}\frac{\widehat{PNpt}_{srow}}{\widehat{PNtt}_{srow}\ \widehat{Pdti}_{srow}{}^2}yrow_{srow} \tag{98}$$

The bounds on the Reynolds numbers are:

$$\frac{\widehat{ms}}{\widehat{\mu s}}\sum_{srow}\frac{\widehat{PDeq}_{srow}(\widehat{PNb}_{srow}+1)}{\widehat{PDs}_{srow}\ \widehat{PFAR}_{srow}\ \widehat{PL}_{srow}}yrow_{srow} \geq 2\cdot 10^3 \tag{99}$$

$$\frac{4\ \widehat{mt}}{\pi\ \widehat{\mu t}}\sum_{srow}\frac{\widehat{PNpt}_{srow}}{\widehat{PNtt}_{srow}\ \widehat{Pdti}_{srow}}yrow_{srow} \geq 10^4 \tag{100}$$

5.4. Geometric Constraints. The maximum and minimum baffle spacing constraints are:

$$\sum_{srow}\frac{\widehat{PL}_{srow}}{(\widehat{PNb}_{srow}+1)}\ yrow_{srow} \leq\ 1.0\ \sum_{srow}\widehat{PDs}_{srow}\ yrow_{srow} \tag{101}$$

$$\sum_{srow}\frac{\widehat{PL}_{srow}}{(\widehat{PNb}_{srow}+1)}\ yrow_{srow} \geq\ 0.2\ \sum_{srow}\widehat{PDs}_{srow}\ yrow_{srow} \tag{102}$$

The constraints associating the ratio between the tube length and the shell diameter are:

$$\sum_{srow}\widehat{PL}_{srow}\ yrow_{srow} \leq\ 15\ \sum_{srow}\widehat{PDs}_{srow}\ yrow_{srow} \tag{103}$$

$$\sum_{srow} \widehat{PL}_{srow}\, yrow_{srow} \geq 3 \sum_{srow} \widehat{PDs}_{srow}\, yrow_{srow} \tag{104}$$

5.5. Objective Function. The expression of the objective function in relation to the binary variables is given by:

$$\text{Min } \pi \sum_{srow} \widehat{PNtt}_{srow}\, \widehat{Pdte}_{srow}\, \widehat{PL}_{srow} yrow_{srow} \tag{105}$$

5.6. Additional Constraints for the Reduction of the Search Space. These extra sets of constraints aim to accelerate the search and are derived from the bounds on velocities, shell-side pressure drop, and tube length/shell diameter ratio. A lower bound on the heat transfer area is also included based on maximum flow velocities (see Gonçalves et al. (2016) for further details).

Flow velocities Bounds.

$$yrow_{srow} = 0 \qquad \text{for } (srow) \in (Svsminout \cup Svsmaxout) \tag{106}$$

$$yrow_{srow} = 0 \qquad \text{for } (srow) \in (Svtminout \cup Svtmaxout) \tag{107}$$

The sets *Svsminout*, *Svsmaxout*, *Svtminout*, and *Svtmaxout* are given by:

$$Svsminout = \{(srow) \,/\, \frac{\widehat{ms}}{\widehat{\rho s}} \frac{(\widehat{PNb}_{srow}+1)}{\widehat{PDs}_{srow}\, \widehat{PFAR}_{srow}\, \widehat{PL}_{srow}} \leq \widehat{vsmin} - \varepsilon\} \tag{108}$$

$$Svsmaxout = \{(srow) \,/\, \frac{\widehat{ms}}{\widehat{\rho s}} \frac{(\widehat{PNb}_{srow}+1)}{\widehat{PDs}_{srow}\, \widehat{PFAR}_{srow}\, \widehat{PL}_{srow}} \geq \widehat{vsmax} + \varepsilon\} \tag{109}$$

$$Svtminout = \{(srow) \,/\, \frac{4\, \widehat{mt}}{\pi\, \widehat{\rho t}} \frac{\widehat{PNpt}_{srow}}{\widehat{PNtt}_{srow}\, \widehat{Pdti}_{srow}^{\;2}} \leq \widehat{vtmin} - \varepsilon\} \tag{110}$$

$$Svtmaxout = \{(srow) \,/\, \frac{4\, \widehat{mt}}{\pi\, \widehat{\rho t}} \frac{\widehat{PNpt}_{srow}}{\widehat{PNtt}_{srow}\, \widehat{Pdti}_{srow}^{\;2}} \geq \widehat{vsmax} + \varepsilon\} \tag{111}$$

where ε is a small positive number.

Shell-side Pressure Upper Bound.

$$yrow_{srow} = 0 \qquad \text{for } (srow) \in SDPsmaxout \tag{112}$$

where the set *SDPsmaxout* is given by:

$$SDPsmaxout = \{(srow) \;/\; \widehat{P\Delta Ps}_{srow} \geq \widehat{\Delta Psdisp} + \varepsilon\} \tag{113}$$

Baffle Spacing.

$$yrow_{srow} \leq 0 \qquad \text{for } (srow) \in (SLNbminout \cup SLNbmaxout) \tag{114}$$

where the sets *SLNbminout* and *SLNbmaxout* are given by:

$$SLNbminout = \{(srow) \;/\; \frac{\widehat{PL}_{srow}}{\widehat{PNb}_{srow}+1} \leq 0.2\widehat{PDs}_{srow} - \varepsilon\} \tag{115}$$

$$SLNbmaxout = \{(srow) \;/\; \frac{\widehat{PL}_{srow}}{\widehat{PNb}_{srow}+1} \geq 1.0\widehat{PDs}_{srow} + \varepsilon\} \tag{116}$$

Tube length / shell diameter ratio.

$$yrow_{srow} \leq 0 \text{ for } (srow) \in (SLDminout \cup SLDmaxout) \tag{117}$$

where the sets *SLDminout* and *SLDmaxout* are given by:

$$SLDminout = \{(srow) \;/\; \widehat{PL}_{srow} \leq 3\widehat{PDs}_{srow} - \varepsilon\} \tag{118}$$

$$SLDmaxout = \{(srow) \;/\; \widehat{PL}_{srow} \geq 15\widehat{PDs}_{srow} + \varepsilon\} \tag{119}$$

Heat Transfer Area.

$$yrow_{srow} = 0 \qquad \text{for } (srow) \in SAminout \tag{120}$$

where the set of heat exchangers with area lower than the minimum possible is:

$$SAminout = \{(srow) \;/\pi\; \widehat{PNtt}_{srow}\widehat{Pdte}_{srow}\widehat{PL}_{srow} \leq \widehat{Amin} - \varepsilon\} \tag{121}$$

The lower bound on the heat transfer area can be determined by:

$$\widehat{Amin} = \frac{\hat{Q}}{\widehat{Umax}\widehat{\Delta Tlm}} \tag{122}$$

$$\widehat{Umax} = \frac{1}{\frac{1}{\widehat{htmax}}\widehat{drmin} + \widehat{Rft}\cdot\widehat{drmin} + \frac{\widehat{Pdte}_{srow}\ln(\widehat{drmin})}{2\,\widehat{ktube}} + \widehat{Rfs} + \frac{1}{\widehat{hsmax}}} \tag{123}$$

$$\widehat{htmax} = \max(\widehat{Pht}_{srow}) \tag{124}$$

$$\widehat{hsmax} = \max(\widehat{Phs}_{srow}) \tag{125}$$

$$\widehat{drmin} = \min(\widehat{Pdte}_{srow}/\widehat{Pdti}_{srow}) \tag{126}$$

6. RESULTS

The five aggregation alternatives of the discrete variables were applied to the sample of ten thermal tasks proposed by Gonçalves et al. (2016), involving different heating and cooling services. Table 2 displays the description of each problem, and Tables 3 and 4 show the hot and cold streams data. The standard values of the discrete variables employed in the solutions are shown in Table 5, related to a fixed tube-sheet type exchanger with tube thickness of 1.65 mm (BWG 16) and thermal conductivity of the tube wall equal to 50 W/m K. The minimum excess area is 11% and the tube count data is based on Kakaç et al. (2012).

Table 2. Design Tasks

Example	1	2	3	4	5
Service	Crude oil cooler	Crude oil cooler	Methanol cooler	Methanol cooler	Methanol heater
Hot stream	Crude oil	Crude oil	Methanol	Methanol	Hot water
Cold stream	Cooling water	Cooling water	Cooling water	Cooling water	Methanol
Tube-side stream	Cold	Cold	Hot	Hot	Hot

Example	6	7	8	9	10
Service	Ethanol cooler	Sucrose solution heater	Sucrose solution cooler	Acetone ethanol exchanger	Acetone ethanol exchanger
Hot stream	Ethanol	Hot water	Sucrose solution	Ethanol	Ethanol
Cold stream	Cooling water	Sucrose solution	Cooling water	Acetone	Acetone
Tube-side stream	Cold	Hot	Cold	Cold	Hot

Table 3. Hot Stream Data

Example	1	2	3	4	5	6	7	8	9	10
$\widehat{m}$ (kg/s)	110.0	50.0	27.8	69.4	40.0	55.6	40.0	83.3	111.1	111.1
Inlet $\widehat{T}$ (°C)	90.0	100.0	70.0	100.0	220.0	150.0	220.0	90.0	190.0	190.0
Outlet $\widehat{T}$ (°C)	50.0	50.0	40.0	40.0	110.2	60.0	80.8	40.0	120.0	120.0
max ΔP (kPa)	100	60	70	70	70	70	70	100	100	100
ρ (kg/m^3)	786	786	750	750	888	789	888	1080	789	789
$\widehat{\mu}$ (mPa·s)	1.89	1.89	0.34	0.34	0.15	0.67	0.15	1.30	0.67	0.67
$\widehat{cp}$ (J/kg·K)	2177	2177	2840	2840	4312	2470	4312	3601	2470	2470
$\widehat{k}$ (W/m·K)	0.12	0.12	0.19	0.19	0.70	0.17	0.70	0.58	0.17	0.17
$\widehat{Rf}$ (m^{2}K/W)	0.0002	0.0002	0.0002	0.0002	0.0001	0.0002	0.0001	0.0001	0.0002	0.0002

Table 4. Cold Stream Data

Example	1	2	3	4	5	6	7	8	9	10
$\hat{m}$ (kg/s)	228.8	130.0	56.6	353.3	133.3	295.0	133.3	358.3	166.7	166.7
Inlet $\hat{T}$ (°C)	30.0	30.0	30.0	32.0	30.0	30.0	30.0	30.0	30.0	30.0
Outlet $\hat{T}$ (°C)	40.0	40.0	40.0	40.0	80.0	40.0	80.0	40.0	79.7	79.7
max ΔP (kPa)	100	50	100	70	70	70	70	100	100	100
ρ (kg/m^3)	995	995	995	995	750	995	1080	995	736	736
$\hat{\mu}$ (mPa·s)	0.72	0.72	0.72	0.72	0.34	0.72	1.30	0.80	0.21	0.21
$\widehat{cp}$ (J/kg·K)	4187	4187	4187	4187	2840	4187	3601	4187	2320	2320
$\hat{k}$ (W/m·K)	0.59	0.59	0.59	0.59	0.19	0.59	0.58	0.59	0.14	0.14
$\widehat{Rf}$ (m^{2}K/W)	0.0004	0.0003	0.0002	0.0004	0.0001	0.0004	0.0001	0.0004	0.0002	0.0002

Table 5. Standard Values of the Discrete Design Variables

Variable	Values
Outer tube diameter $\widehat{pdte}_{sd}$ (m)	0.019, 0.025, 0.032, 0.038, 0.051
Tube length, $\widehat{pL}_{sL}$ (m)	1.220,1.829,2.439, 3.049, 3.659, 4.877, 6.098
Number of baffles, $\widehat{pNb}_{sNb}$	1, 2, … , 20
Number of tube passes, $\widehat{pNpt}_{sNpt}$	1, 2, 4, 6
Tube pitch ratio, $\widehat{prp}_{srp}$	1.25, 1.33, 1.50
Shell diameter, $\widehat{pDs}_{sDs}$ (m)	0.787, 0.838, 0.889, 0.940,0. 991, 1.067, 1.143, 1.219, 1.372, 1.524
Tube layout, $\widehat{play}_{slay}$	1 = square, 2 = triangular

These problems were solved using the five alternatives of MILP formulations described in Table 1, implemented in the optimization software GAMS using the solver CPLEX.

The comparison of the solution time demanded by each alternative (elapsed time) and the time consumed by the solver itself are shown in Table 6, together with the optimal value of the objective function (since all alternatives are MILP problems, the solution found is always the same, corresponding to the global optimum). The computational times were measured using a computer with a processor Intel Core i7 3.40 GHz with 12.0 GB RAM memory.

Table 6. Performance Comparison

		solution time (s) solver time (s)				
example	heat transfer area (m^2)	1	2	3	4	5
1	624	1730 1726	19 13	13 6	11 4	12 3
2	319	1574 1571	44 39	12 6	10 4	11 3
3	199	212 208	10 5	11 5	9 3	11 3
4	872	139 136	11 5	11 5	9 3	12 3
5	144	869 865	19 13	11 6	10 4	12 3
6	332	2755 2751	49 44	12 6	9 4	11 3
7	207	2535 2532	15 9	12 5	9 4	12 3
8	914	173 169	11 6	13 7	9 3	12 3
9	287	2077 2073	53 47	31 25	11 6	12 3
10	327	2342 2338	43 37	25 19	10 4	12 3
Average	-	1458.4	27.4 21.8	15.1 9.0	9.7 3.9	11.7 3.0

The solution times in Table 6 for the Alternative 1 differs slightly of those reported in Gonçalves et al. (2016) due to eventual computer performance fluctuations (the registered times are wall times from new independent runs conducted in this paper for the same problems).

The analysis of Table 6 indicates that the proposed procedure of aggregation of the binary variables (Alternatives 2 to 5) allows large reductions of the computational effort in relation to the original formulation (Alternative 1). The average time consumed by the solver is associated to reductions ranging from 98.50% to 99.79%. The corresponding reductions of the total elapsed time are similar, ranging from 98.1% to 99.33%.

Comparing the time consumed by the solver in the different alternatives, it is possible to observe that there is a reduction trend from Alternative 1 to Alternative 5, i.e. the increase of the binary variables aggregation decreases the solver time. The behavior of the total elapsed time is similar, but the demand for processing larger data sets associated to the variable aggregation procedure implies in slightly higher computing times before the solver starts in these alternatives. Therefore, the lowest solver times is associated to the Alternative 5, but the lowest elapsed times correspond to Alternative 4 (however, the difference is only 2 s).

7. CONCLUSIONS

This paper presented an investigation aiming at the reduction of the computational effort for the solution of the MILP problem for the design of shell and tube heat exchangers. Several MILP formulations were proposed based on different alternatives of aggregation of the discrete values of the design variables in relation to the binary variables.

In the original paper of Gonçalves et al. (2016), where the MILP formulation was proposed, each discrete value corresponds to a binary variable. The alternatives developed in this paper tried to aggregate the discrete alternatives in tables, where each group of discrete values becomes an individual binary variable.

The results showed that the aggregation of the binary variables allows a considerable reduction of the computational effort to solve the MILP problem. Considering a sample of 10 design problems, the best aggregation alternative demanded only 0.21% of the total solver time in comparison of the original MILP.

This performance gain is important because allows further investigations for the inclusion of this model into more complex problems, such as, the insertion of the detailed heat exchanger design into the heat exchanger network synthesis problem.

AUTHOR INFORMATION

Corresponding Author

*E-mail: bagajewicz@ou.edu

Notes

The authors declare no competing financial interest.

NOMENCLATURE

Sets

sd = tube diameter, 1 , … , $sdmax$

sDs = shell diameter, 1 , … , $sDsmax$

sL = tube length, 1 , … , $sLmax$

$slay$ = tube layout, 1… $slaymax$

sNb = number of baffles, 1, … , $sNbmax$

$sNpt$ = number of tube passes, 1 , … , $sNptmax$

srp = tube pitch ratio, 1 , … , $srpmax$

$srow$ = multi-index

Parameters

$\widehat{Aexc}$ = excess area, %

$\widehat{cp}$ = heat capacity, J/kg K

$\hat{g}$ = gravity acceleration, m/s2

$\hat{k}$ = thermal conductivity, W/m K

$\hat{m}$ = mass flow rate, kg/s

$\hat{n}$ = 0.4 for heating services; 0.3 for cooling services

$\hat{P}$ = LMTD correction factor parameter

$\widehat{PDeq}_{srp,sd,slay}$ = equivalent diameter, m

$\widehat{PDs}_{sDs}$ = shell diameter, m

$\widehat{Pdte}_{sd}$ = outer tube diameter, m

$\widehat{Pdti}_{sd}$ = inner tube diameter, m

$\widehat{PL}_{sL}$ = tube length, m

$\widehat{Play}_{slay}$ = tube layout

$\widehat{PNb}_{sNb}$ = number of baffles

$\widehat{PNpt}_{sNpt}$ = number of tube passes

$\widehat{PNtt}_{srow}$ = total number of tubes

$\widehat{Prp}_{srp}$ = tube pitch ratio

$\widehat{Pr}$ = Prandtl number

$\hat{Q}$ = heat duty, W

$\hat{R}$ = LMTD correction factor parameter

$\widehat{Rf}$ = fouling factor, m^2 K/W

$\hat{T}$ = temperature, ^{o}C

$\hat{\rho}$ = density, kg/m^3

$\hat{\mu}$ = viscosity, Pa·s

$\widehat{\Delta Pdisp}$ = available pressure drop, Pa

$\widehat{\Delta Tlm}$ = log-mean temperature difference

Binary variables

yd_{sd} = variable representing the tube diameter

yDs_{sDs} = variable representing the shell diameter

yL_{sL} = variable representing the tube length

$ylay_{slay}$ = variable representing the tube layout

yNb_{sNb} = variable representing the number of baffles

$yNpt_{sNpt}$ = variable representing the number of tube passes

yrp_{srp} = variable representing the tube pitch ratio

$yrow_{srow}$ = variable representing the set of variables

Continuous variables

A = area, m^2

Ar = flow area in the shell side, m^2

d = tube diameter, m

Deq = 30quivalente diameter, m

Ds = shell diamenter, m

f = friction factor

F = correction factor to logarithmic mean temperature difference

h = convective heat transfer coefficient, W/m^2 K

K = pressure drop parameter

L = tube length, m

lbc = baffle spacing, m

ltp = tube pitch, m

Nb = number of baffles

Npt = number of tube passes

Ntp = number of tubes per passes

Ntt = total number of tubes

Nu = Nusselt number

Re = Reynolds number

rp = tube pitch ratio

U = overall heat transfer coefficient, W/m^2 K

v = velocity, m/s

ΔP = pressure drop, Pa

Subscripts

c = cold fluid

h = hot fluid

i = inlet

o = outlet

s = shell-side

t = tube-side

$tube$ = heat exchanger tube variable

max = maximum value

min = minimum value

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