# Design of Multi-echelon Supply Chain Networks under Demand Uncertainty

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We consider the design of multiproduct, multi-echelon supply chain networks. The networks comprise a number of manufacturing sites at fixed locations, a number of warehouses and distribution centers of unknown locations (to be selected from a set of potential locations), and finally a number of customer zones at fixed locations. The system is modeled mathematically as a mixed-integer linear programming optimization problem. The decisions to be determined include the number, location, and capacity of warehouses and distribution centers to be set up, the transportation links that need to be established in the network, and the flows and production rates of materials. The objective is the minimization of the total annualized cost of the network, taking into account both infrastructure and operating costs. A case study illustrates the applicability of such an integrated approach for production and distribution systems with or without product demand uncertainty.

## 1. Introduction

A supply chain is defined as a network of facilities that performs the functions of procurement of materials, transformation of these materials into intermediate and finished products, and distribution of these products to customers.[1] A similar definition has been given by Bhaskaran and Leung,[2] who describe the manufacturing supply chain as an integrative approach used to manage the inter-related flows of products and information among suppliers, manufacturers, distributors, retailers, and customers.

A typical supply chain (see Figure 1) comprises suppliers, production sites, storage facilities, and customers. It involves two basic processes tightly integrated with each other: (i) the production planning and inventory control process, which deals with manufacturing, storage, and their interfaces, and (ii) the distribution and logistics process, which determines how products are retrieved and transported from the warehouse to retailers.

Suppliers are at the start of the supply chain providing raw material to the manufacturers. Each manufacturer may have more than one supplier.

The manufacturing sites of interest to this paper are multipurpose production plants where a wide range of products can be produced. The production capacity of each site is typically determined by the detailed scheduling of each plant.

Before being distributed to the customers, the final products from production plants are stored at two distinct stages in the supply chain, namely, at major warehouses and at smaller distribution centers. Each warehouse may be supplied material from more than one manufacturing site. Similarly, a distribution center can be supplied from more than one warehouse although, for reasons of organizational simplicity (“single sourcing”), it is often the case that each distribution center is supplied by only one warehouse. Both the material storage and handling capacities of warehouses and distribution centers are limited within certain bounds.

At the end of the supply chain, there are the customers. Usually, each customer is assigned to a single distribution center which supplies all of the required material, although this may not always be the case. Customers place their orders at distribution centers which pass this information to the upper levels until it gets to the suppliers. Thus, a main characteristic of the supply chain is the flow of material from suppliers to customers and the counterflow of information from customers to suppliers.[3]

The places where inventory is kept in the supply chain are called “echelons”. Usually the complexity of a supply chain is related to the number of echelons that it incorporates.

The operation of supply chains is a complex task because of the large-scale physical production and distribution network flows, the uncertainties associated with the external customer and supplier interfaces, and the nonlinear dynamics associated with internal information flows. In a highly competitive environment, a supply chain should be managed in the most efficient way, with the objectives of (i) *minimization* of costs, delivery delays, inventories, and investment (ii) *maximization* of deliveries, profit, return on investment (ROI), customer service level, and production.

The above tasks involve both strategic and operational decisions, with time horizons ranging from several years down to a few hours, respectively:[1]

1. *Location decisions* consider the number, size, and physical location of production plants, warehouses, and distribution centers.

2. *Production decisions* consider the products to be produced at each plant and also the allocation of suppliers to plants, of plants to distribution centers, and of distribution centers to customers.

The detailed production scheduling at each plant must also be decided.

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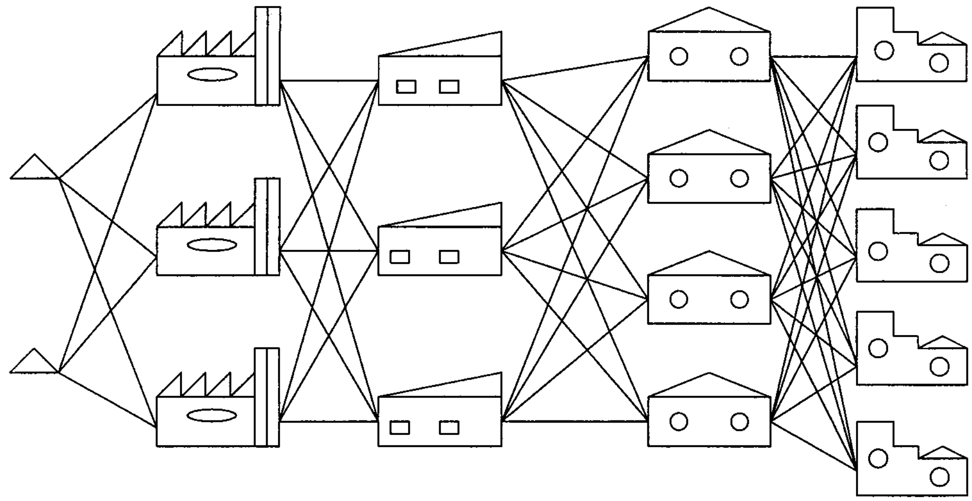


**Figure 1.** Typical supply chain network.

3. *Inventory decisions* are concerned with the management of the inventory levels.

4. *Transportation decisions* include the transportation media to be used for and the size of each shipment of material.

As supply chains become increasingly global,[5] additional aspects such as differences in tax regimes, duty drawback and avoidance, and fluctuations in exchange rates also become important.

Section 2 of this paper presents a critical review of past work on supply chain modeling and design. In section 3, the problem of optimal design of supply chains is formulated as a mixed-integer linear programming (MILP) model. The latter includes some of the features that earlier models have failed to consider. Section 4 extends this formulation to take into account the uncertainty in customer demands. A case study is presented in section 5 to illustrate the applicability of the model.

## 2. Literature Review

In view of their importance in the modern economy, it is not surprising that supply chains have been on the research agenda of a variety of business and other academic disciplines for many years.

The review presented in this section focuses on model-based methodologies that can provide *quantitative* support to the design and operation of supply chains. The major desicions that need to be made in this context have already been described in the introduction of this paper.

**2.1. Heuristic-Based Approaches.** Williams[6] presents seven heuristic algorithms for scheduling production and distribution operations in supply chain networks, comparing them with each other and with a dynamic programming model. The objective is to determine a minimum cost production and product distribution schedule, satisfying the product demand, in a given distribution network. It is assumed that the demand rate is constant and that processing is instantaneous, with no delivery lags between facilities.

The risks arising from the use of heuristics in distribution planning were identified and discussed early on by Geoffrion and Van Roy.[7] They pointed out that heuristics (computerized or not) can lead to wrong decisions. Three examples were presented in the area of distribution planning considering questions such as the number, size, and location of plants and distribution facilities, the stocks, and the policies regarding inventory and transportation. All three examples demonstrate the failure of "common sense" methods to come up with the best possible solution.

**2.2. Mathematical Programming-Based Approaches.** The alternative to heuristics is the use of mathematical models of supply chains. Such models can be classified into four categories:[3] (1) deterministic, (2) stochastic, (3) economic, and (4) simulation.

Workable and realistic models and algorithms began to emerge in the mid-1970s with the emerging improvements in computational capabilities.

In their pioneering work, Geoffrion and Graves[8] present a model to solve the problem of designing a distribution system with optimal location of the intermediate distribution facilities between plants and customers. The objective is to minimize the total distribution cost (including transportation cost and investment cost), subject to a number of constraints such as supply constraints, demand constraints, and specification constraints regarding the nature of the problem. The problem is formulated as an MILP, which is solved using Benders decomposition.

Wesolowsky and Truscott[9] present a mathematical formulation for the multiperiod location−allocation problem with relocation of facilities. They model a small distribution network comprising a set of facilities aiming to serve the demand at given points.

Williams[10] develops a dynamic programming algorithm for simultaneously determining the production and distribution batch sizes at each node within a supply chain network. The average cost is minimized over an infinite horizon.

Another deterministic model presented by Ishii et al.[11] aims to calculate the base stock levels and lead times associated with the lowest cost solution for an integrated supply chain in a finite time horizon.

Brown et al.[12] present an optimization-based algorithm for a decision support system used to manage complex problems involving facility selection, equipment location and utilization, and manufacture and distribution of products. They focus on operational issues such as where each product should be produced, how much should be produced in each plant, and from which plant products should be shipped to customers. Some strategic issues are also taken into account such as the number, kind, and location of facilities (including plants).

Breitman and Lucas[13] present a modeling system named PLANETS (Production Location Analysis Network System). This program is used to decide what products to produce and when, where, and how to make these products. It also provides information on shipping allocations, capital spending schedules, and resource usage.

An MINLP formulation is presented by Cohen and Lee,[14] seeking to maximize the total after-tax profit for the manufacturing facilities and distribution centers. Managerial (resource and production) as well as logical consistency (feasibility, availability, and demands limits) constraints were applied. This work was extended by Cohen and Moon,[15] who developed a constrained optimization model, called PILOT, to investigate the effects of various parameters on supply chain cost and to determine which manufacturing facilities and distribution centers should be established.

A two-phase approach was used by Newhart et al.[16] to design an optimal supply chain. First, a combination of mathematical programming and heuristic models is used to minimize the number of product types held in inventory throughout the supply chain. In the second phase, a spreadsheet-based inventory model determines the minimum safety stock required to absorb demand and lead time fluctuations.

Chandra[17] presents a model that plans deliveries to customers based upon inventories, at warehouses and distribution centers, and vehicle routes. In a later work, Chandra and Fisher[18] consider the coordination of production and distribution planning.

Pooley[19] presents the results of an MILP formulation used by the Ault Foods company to restructure their supply chain. The model aims to minimize the total operating cost of a production and distribution network. The model is used to answer questions like the following: Where should the division locate plants and depots (DCs)? How should the production be allocated? How should the customers be served?

Arntzen et al.[20] developed an MILP “global supply chain model” (GSCM) aiming to determine: (1) the number and location of distribution centers, (2) customer−distribution center assignment, (3) number of echelons, and (4) the product−plant assignment. The objective of the model is to minimize a weighted combination of total cost (including production, inventory, transportation, and fixed costs) and activity days.

Voudouris[21] developed a mathematical model designed to improve the efficiency and responsiveness in a supply chain. The target is to improve the flexibility of the system. He identifies two types of manufacturing resources: activity resources (manpower, warehouse doors, packaging lines, forktrucks, etc.) and inventory resources (volume of intermediate storage, warehouse area, etc.). The objective function aims to represent the flexibility of the plant to absorb unexpected demands.

Camm et al.[22] developed an integer programming model used by the Procter and Gamble company to determine the location of distribution centers and to assign those selected to customer zones. A variety of reasons (including the need to reduce transportation costs, the introduction of new products, new quality policies, the reduction in the life cycle of products, and relatively high number of plants) forced the company to restructure their supply chain. The models developed were a simple transportation model and an uncapacitated facility-location problem, formulated as LP and MILP models, respectively.

Pirkul and Jayarama[23] studied a tri-echelon multicommodity system concerning production, transportation, and distribution planning. The objective is to minimize the combined costs of establishing and operating the plants and the warehouses in addition to any transportation and distribution costs from plants to warehouses and from warehouses to customers. Key decisions are where to locate warehouses and which customers to assign to each warehouse. Another decision includes the selection of manufacturing plants and their assignment to warehouses. Lagrangian relaxation is used to solve the problem which is formulated as an MILP.

In a later work, Pirkul and Jayarama[24] present the PLANWAR model that seeks to locate a number of production plants and distribution centers so that the total operating cost for the distribution network is minimized. The network consists of a potential set of production plants and distribution centers and a set of given customers. This model has many similarities to the one presented by the same authors in 1996.

Hindi and Pienkosz[25] present a solution procedure for large-scale, single-source, capacitated plant location problems (SSCPLP). They present an MILP formulation of the problem, with two types of decision variables relating to the selection of plants and to the allocation of customers to plants, respectively.

In addition to the above works, there is also a substantial literature on the problem of facility location. Current et al.[26] review the work in this area up to 1990. They consider four categories of objectives: (1) cost minimization, (2) demand satisfaction, (3) profit maximization, and (4) environmental concerns. Issues of interest are also the number and types of facilities being sited, their capacities, the nature of the problem (continuous or discrete), the nature of the parameters (stochastic or deterministic), and finally the nature of the model (static or dynamic). More recent work is reviewed by Owen and Daskin,[27] while Sridharan[28] reviews the solution methods for such problems.

This paper proposes a strategic planning model for multi-echelon supply chain networks, integrating components associated with production, facility location, product transportation, and distribution. It considers multiproduct, multi-echelon supply chain networks operating under uncertainty. The network comprises a number of manufacturing sites, each using a set of flexible, shared resources for the production of a number of products. The manufacturing sites are assumed already to exist at given locations and so do the customer zones. The potential establishment of a number of potential warehouses and distributions centers at locations to be selected from a set of possible candidates is considered as part of the optimization.

The above problem is formulated mathematically as an MILP optimization problem and is solved using branch-and-bound techniques. Thus, in addition to providing a single model that integrates all of the aspects of the supply chain mentioned above, our aim is to obtain an optimal design of such systems. In view of the large money flows involved in supply chain networks, there may be substantial differences between optimal and suboptimal solutions.

Compared to the other models that have been presented in the literature to date (see Table 1), our model integrates three distinct echelons of the supply chain, within a single, mathematical programming-based formulation. Moreover, it takes into account the complexity introduced by the multiproduct nature of the production facilities, the economies of scale in transportation, and the uncertainty inherent in the product demands.

## 3. Optimal Steady-State Design of Supply Chain Networks

This work considers the design of multiproduct, multiechelon production and distribution networks. As shown in Figure 2, the network consists of a number of existing multiproduct manufacturing sites at fixed locations, a number of warehouses and distribution centers of unknown locations (to be selected from a set of candidate locations), and finally a number of customer zones at fixed locations.

In general, each product can be produced at several plants at different locations. The production capacity of each manufacturing site is modeled in terms of a set of linear constraints relating the mean production rates of the various products to the availability of one or more production resources. Warehouses and distribution centers are described by upper and lower bounds on their material handling capacity. Warehouses can be supplied from more than one manufacturing site and can supply more than one distribution center. Each distribution center can be supplied by more than one warehouse. However, "single sourcing" constraints requiring that a distribution center be supplied by a single warehouse (to be determined by the optimization) are also accommodated.

Each customer zone places demands for one or more products. These demands may be assumed to be known a priori; alternatively, a number of demand scenarios, each with a given, nonzero probability, may be considered. A customer may be served by more than one distribution center. Alternatively, single sourcing constraints, according to which a customer zone must be served by a single distribution center (to be determined by the optimization), may be imposed.

The establishment of warehouses and distribution centers incurs a fixed infrastructure cost. Operational costs include those associated with production, handling of material at warehouses and distribution centers, and transportation. Transportation costs are assumed to be piecewise linear functions of the actual flow of the product from the source stage to the destination stage, and they may include taxes and duties.

The decisions to be determined include the number, location, and capacity of warehouses and distribution centers to be set up, the transportation links that need to be established in the network, and the flows and production rates of materials. The objective is the minimization of the total annualized cost of the network, taking into account both infrastructure and operating costs.

This paper considers a steady-state form of the above problem according to which demands are time-invariant (but possibly uncertain) and all production and transportation flows determined by the optimization are considered to be time-averaged quantities. Section 3 considers the deterministic case of known product

**Table 1. Supply Chain Models Reviewed in This Work**

| | | model type | | | model features | | | operational decisions | | strategic decisions | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ref | no. of echelons considered[a] | heuristic | deterministic | stochastic | production model[b] | transportation cost model[c] | uncertainty | production assignment to plant | inventory levels | warehouse−DC assignment/location | DC−customer assignment/location | objective function |
| 6 | no echelons | * | | | MP | C | | * | | | | cost |
| 8 | DC | | * | | | C | | * | * | | * | cost |
| 10 | no echelons | | * | | MP | C | | * | | | | cost |
| 12 | no echelons | | * | | | C | | * | | | * | cost |
| 14 | P, W | | * | | MP | C | | * | * | | * | cost |
| 15 | no echelons | | * | | MP | C | | * | | | | cost |
| 17 | not clear | | * | | | C | | | * | | | cost |
| 18 | not clear | | * | | | C | | | * | | | cost |
| 19 | not clear | | * | | | C | | * | | | * | cost |
| 20 | not clear | | * | | | C | | * | * | | * | cost |
| 21 | P | | * | | | C | | * | * | | | flexibility |
| 22 | no echelons | | * | | | C | | | | | * | cost |
| 23 | P, W | | * | | | C | | * | | | * | cost |
| 24 | W | | * | | | C | | * | | | * | cost |
| 25 | no echelons | | * | | | C | | * | | | * | cost |
| 26 | no echelons | | * | | | C | | | | * | * | cost |
| 31 | P, W | | | * | MP | C | * | | * | | | cost |
| 32 | P, W | | | * | | C | * | | * | | | cost/response |
| 33 | P | | | * | | C | * | | * | | | customer response |
| 34 | P | | | * | MP | C | * | | * | | | cost |
| 35 | P | | | * | MP | C | * | | * | | | cost |
| proposed model | P, W, DC | | * | | MP | V | * | * | * | * | * | cost |

[a] Places where inventory is held: P, plant; W, warehouse; DC, distribution center. [b] SP: single product. MP: multiproduct. [c] C: constant. V: variable.

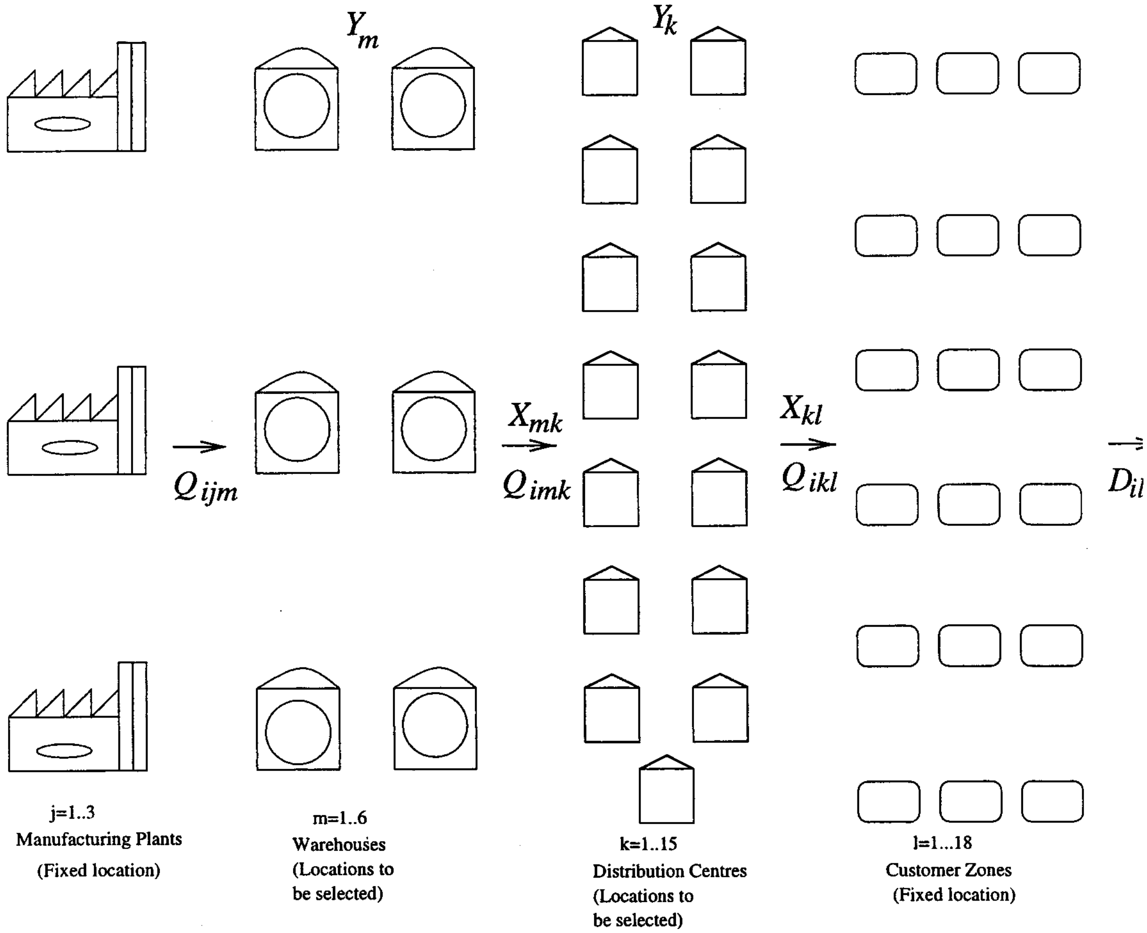


**Figure 2.** Overview of the supply chain network.

demands, formulating the problem as an MILP. Section 4 extends the formulation to the case of uncertain demands described in terms of multiple scenarios.

**3.1. Variables. 3.1.1. Binary Variables.** Four main types of binary variables are defined:

$$Y_m = \begin{cases} 1, & \text{if the warehouse at candidate position } m \text{ is to be established} \\ 0, & \text{otherwise} \end{cases}$$

$$Y_k = \begin{cases} 1, & \text{if the distribution center at candidate position } k \text{ is to be established} \\ 0, & \text{otherwise} \end{cases}$$

$$X_{mk} = \begin{cases} 1, & \text{if warehouse } m \text{ is to supply distribution center } k \\ 0, & \text{otherwise} \end{cases}$$

$$X_{kl} = \begin{cases} 1, & \text{if distribution center } k \text{ is to supply customer zone } l \\ 0, & \text{otherwise} \end{cases}$$

**3.1.2. Continuous Variables.** This formulation uses a number of continuous variables to describe the network: (i) $P_{ij}$ is the rate of production of product $i$ by plant $j$. (ii) $Q_{ijm}$ is the rate of flow of product $i$ from plant $j$ to warehouse $m$. (iii) $Q_{imk}$ is the rate of flow of product $i$ from warehouse $m$ to distribution center $k$. (iv) $Q_{ikl}$ is the rate of flow of product $i$ from distribution center $k$ to customer zone $l$. These quantities are not generally known a priori; in fact, their optimal values will depend strongly on the product demands, the production capacity of the various plants, and the transportation costs.

Each product is stored twice before reaching the customer, first in a warehouse and then in a distribution center. The capacity of both has to be determined as part of the design of the distribution network. Therefore, we introduce the following variables: (i) $W_m$ is the capacity of warehouse $m$. (ii) $D_k$ is the capacity of distribution center $k$.

**3.2. Constraints. 3.2.1. Network Structure Constraints.** A link between a warehouse $m$ and a distribution center $k$ can exist only if warehouse $m$ also exists:

$$X_{mk} \le Y_m, \quad \forall\ m, k \tag{1}$$

If it is required that a certain distribution center be served by a single warehouse, then this can be enforced via the constraint

$$\sum_m X_{mk} = Y_k, \quad \forall\ k \in K^{ss} \tag{2}$$

where $K^{ss}$ is the set of distribution centers for which single sourcing is required.

If the distribution center does not exist, then its links with warehouses cannot exist either. This leads to the constraint

$$X_{mk} \le Y_k, \quad \forall\ m, k \notin K^{ss} \tag{3}$$

We note that the above is written only for distribution centers that are not single-sourced. For single-sourced distribution centers, constraint (2) already suffices.

The link between a distribution center $k$ and a customer zone $l$ will exist only if the distribution center also exists:

$$X_{kl} \leq Y_k, \quad \forall\, k, l \tag{4}$$

Some customer zones may be subject to a single sourcing constraint requiring that they be served by exactly one distribution center:

$$\sum_k X_{kl} = 1, \quad \forall\, l \in L^{ss} \tag{5}$$

while $L^{ss}$ is the set of customer zones for which single sourcing is required.

**3.2.2. Logical Constraints for Transportation Flows.** A flow of material $i$ from plant $j$ to warehouse $m$ can take place only if warehouse $m$ exists:

$$Q_{ijm} \leq Q_{ijm}^{max} Y_m, \quad \forall\, i, j, m \tag{6}$$

A flow of material $i$ from warehouse $m$ to distribution center $k$ can take place only if the corresponding connection exists:

$$Q_{imk} \leq Q_{imk}^{max} X_{mk}, \quad \forall\, i, m, k \tag{7}$$

A flow of material $i$ from distribution center $k$ to customer zone $l$ can take place only if the corresponding connection exists:

$$Q_{ikl} \leq Q_{ikl}^{max} X_{kl}, \quad \forall\, i, k, l \tag{8}$$

Values for the upper bounds $Q_{ijm}^{max}$, $Q_{imk}^{max}$, and $Q_{ikl}^{max}$ appearing on the right-hand sides of constraints (6)–(8) can be obtained as described in section 3.4.

There is usually a minimum total flow rate of material (of whatever type) that is needed to justify the establishment of a transportation link between two locations in the network. This consideration leads to constraints of the form

$$\sum_i Q_{imk} \geq Q_{mk}^{min} X_{mk}, \quad \forall\, m, k \tag{9}$$

$$\sum_i Q_{ikl} \geq Q_{kl}^{min} X_{kl}, \quad \forall\, k, l \tag{10}$$

concerning the links between a warehouse $m$ and a distribution center $k$ and between a distribution center $k$ and a customer demand zone $l$, respectively.

**3.2.3. Material Balances.** The actual rate of production of product $i$ by plant $j$ must equal the total flow of this product from plant $j$ to all warehouses $m$:

$$P_{ij} = \sum_m Q_{ijm}, \quad \forall\, i, j \tag{11}$$

For steady-state operation, there is no stock accumulation or depletion and, therefore, the total rate of flow of each product $i$ leaving a warehouse or a distribution center must equal the total rate of flow entering this node of the supply chain network:

$$\sum_j Q_{ijm} = \sum_k Q_{imk}, \quad \forall\, i, m \tag{12}$$

$$\sum_m Q_{imk} = \sum_l Q_{ikl}, \quad \forall\, i, k \tag{13}$$

The total rate of flow of each product $i$ received by each customer zone $l$ from the distribution centers must be equal to the corresponding market demand:

$$\sum_k Q_{ikl} = D_{il}, \quad \forall\, i, l \tag{14}$$

**3.2.4. Production Resource.** An important issue in designing the distribution network is the ability of the manufacturing plants to cover the demands of the customers as expressed through the orders received from the warehouses.

The rate of production of each product at any plant cannot exceed certain limits. Thus, there is always a maximum production capacity for any one product; moreover, there is often a minimum production rate that must be maintained while the plant is operating:

$$P_{ij}^{min} \leq P_{ij} \leq P_{ij}^{max}, \quad \forall\, i, j \tag{15}$$

It is common in many manufacturing sites for some resources (equipment, utilities, manpower, etc.) to be used by several production lines and at different stages of the production of each product. This shared usage limits the availability of the resource that can be used for any one purpose as expressed by the following constraint:

$$\sum_i \rho_{ije} P_{ij} \leq R_{je}, \quad \forall\, j, e \tag{16}$$

The coefficient $\rho_{ije}$ expresses the amount of resource $e$ used by plant $j$ to produce a unit amount of product $i$, while $R_{je}$ represents the total rate of availability of resource $e$ at plant $j$.

**3.2.5. Capacity of Warehouses and Distribution Centers.** The capacity of a warehouse $m$ generally has to lie between given lower and upper bounds, $W_m^{min}$ and $W_m^{max}$, provided, of course, that the warehouse is actually established (i.e., $Y_m = 1$):

$$W_m^{min} Y_m \leq W_m \leq W_m^{max} Y_m, \quad \forall\, m \tag{17}$$

Similar constraints apply to the capacities of the distribution centers:

$$D_k^{min} Y_k \leq D_k \leq D_k^{max} Y_k, \quad \forall\, k \tag{18}$$

We generally assume that the capacities of the warehouses and the distribution centers are related linearly to the flows of materials that they handle. This is expressed via the constraints

$$W_m \geq \sum_{i,k} \alpha_{im} Q_{imk}, \quad \forall\, m \tag{19}$$

$$D_k \geq \sum_{i,l} \beta_{ik} Q_{ikl}, \quad \forall\, k \tag{20}$$

where $\alpha_{im}$ and $\beta_{ik}$ are given coefficients.

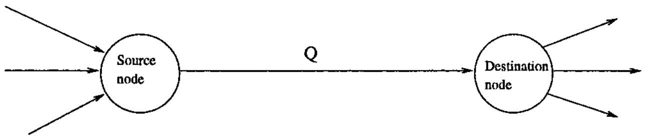


**Figure 3.** Transportation of material in an arc of the supply chain network.

**3.2.6. Nonnegativity Constraints.** All continuous variables must be nonnegative:

$$P_{ij} \geq 0, \quad \forall\ i, j \tag{21}$$

$$Q_{ijm} \geq 0, \quad \forall\ i, j, m \tag{22}$$

$$Q_{imk} \geq 0, \quad \forall\ i, m, k \tag{23}$$

$$Q_{ikl} \geq 0, \quad \forall\ i, k, l \tag{24}$$

**3.3. Objective Function.** In general, a distribution network involves both capital and operating costs. The former are one-off costs associated with the establishment of the infrastructure of the network and, in particular, its warehouses and distribution centers. On the other hand, operating costs are incurred on a daily basis and are associated with the cost of production of material at plants, the handling of material at warehouses and distribution centers, and the transportation of material through the network.

**3.3.1. Fixed Infrastructure Costs.** The infrastructure costs considered by our formulation are related to the establishment of a warehouse or a distribution center at a candidate location. These costs are expressed in the following objective function terms:

$$\sum_m C_m^{\mathrm{W}} Y_m + \sum_k C_k^{\mathrm{D}} Y_k$$

We assume that the production plants are already established. Therefore, we do not consider the capital cost associated with their design and construction. We also ignore any infrastructure cost associated with the customer zones.

**3.3.2. Production Cost.** The production cost incurred at a plant $j$ is assumed to be proportional to the rate of production of each product $i$, with a constant unit production cost $C_{ij}^{\mathrm{P}}$. The corresponding term in the objective function is of the form

$$\sum_{i,j} C_{ij}^{\mathrm{P}} P_{ij}$$

**3.3.3. Material Handling Costs at Warehouses and Distribution Centers.** Handling costs can usually be approximated as linear functions of the throughput of each product being handled. They can be expressed as follows:

$$\sum_{i,m} C_{im}^{\mathrm{WH}} (\sum_j Q_{ijm}) + \sum_{i,k} C_{ik}^{\mathrm{DH}} (\sum_m Q_{imk})$$

**3.3.4. Transportation Costs.** We start by considering generically the transportation cost incurred in transporting a certain flow $Q$ of a material in the arc between any two nodes in the supply chain network, as shown in Figure 3. Usually the *unit* transportation cost is a nonincreasing function of the rate of flow, reflecting economies of scale. Thus, the total transportation cost is a piecewise linear function of the flow of material, of the form shown in Figure 4. The possible range of the transportation flow is divided into NR subranges, each corresponding to a different (and progressively lower) unit transportation price $C_r^{\mathrm{T}}$. The limits of interval $r \in [1, \mathrm{NR}]$ are denoted as $\bar{Q}_{r-1}$ and $\bar{Q}_r$.

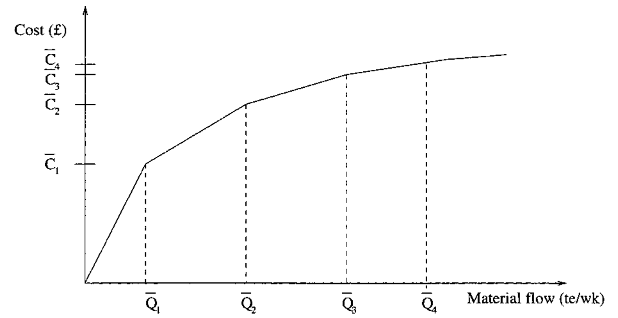


**Figure 4.** Transportation cost as a piecewise linear function of the material flow.

We introduce a new set of binary variables $Z_r$:

$$Z_r = \begin{cases} 1, & \text{if } Q \in [\bar{Q}_{r-1}, \bar{Q}_r] \\ 0, & \text{otherwise} \end{cases}$$

The above definition can be effected via the linear constraints:

$$\bar{Q}_{r-1} Z_r \leq Q_r \leq \bar{Q}_r Z_r, \quad \forall\ r \tag{25}$$

$$\sum_{r=1}^{\mathrm{NR}} Z_r = 1 \tag{26}$$

$$Q = \sum_{r=1}^{\mathrm{NR}} Q_r \tag{27}$$

Constraint (26) ensures that only one of the variables $Z_r$ (say, for $r = r^*$) takes a value of 1, with all others being zero. Constraint (25) then forces $Q_r$ to 0 for all $r \neq r^*$, while also bounding $Q_{r^*}$ in the range $[\bar{Q}_{r^*-1}, \bar{Q}_{r^*}]$. Finally, constraint (27) implies that $Q = Q_r^*$ and, therefore, $Q \in [\bar{Q}_{r^*-1}, \bar{Q}_{r^*}]$, as desired.

The actual transportation cost is given by the linear expression

$$C = \sum_{r=1}^{\mathrm{NR}} \left[ \bar{C}_{r-1} Z_r + (Q_r - \bar{Q}_{r-1} Z_r) \frac{\bar{C}_r - \bar{C}_{r-1}}{\bar{Q}_r - \bar{Q}_{r-1}} \right]$$

We note that, because of constraints (25)−(27), only one of the terms in the above summation will be nonzero, effecting a linear interpolation between two points $(\bar{Q}_{r^*-1}, \bar{C}_{r^*-1})$ and $(\bar{Q}_{r^*}, \bar{C}_{r^*})$ in Figure 4 (where $r^*$ is such that $Z_{r^*} = 1$).

Having established how we can generically model piecewise linear transportation costs, we can apply these manipulations to the flows of material specifically taking place in the supply chain network. Although we could assume that each product $i$ under consideration has a different transportation cost, in reality many of the products in any given supply chain are likely to be very similar to each other. Thus, we introduce the concept of a product family as a subset of the products, all of which have the same unit transportation costs. We denote these families as $I_f$, $f = 1, ..., \mathrm{NF}$, where NF is the number of families. Each product belongs to exactly one family. Thus,

$$\cup_{f=1}^{\mathrm{NF}} I_f = \{i \mid i = 1, ..., \mathrm{NI}\}$$

$$I_f \cap I_{f'} = \emptyset, \quad \forall\, f, f' \in \{1, ..., \mathrm{NF}\}, f \neq f'$$

where NI is the total number of products.

Because all products in a family have the same transportation cost, we can apply the above manipulations to their *combined* flow rate in each arc of the supply chain network. Overall, then, the total transportation cost incurred in the network is given by

$$\sum_{f,j,m} C_{fjm} + \sum_{f,m,k} C_{fmk} + \sum_{f,k,l} C_{fkl} \tag{28}$$

subject to the following constraints:

(i) For the transportation of material between plants and warehouses:

$$\sum_{r=1}^{\mathrm{NR}_{fjm}} Z_{fjmr} = 1, \quad \forall\, f, j, m \tag{29}$$

$$\bar{Q}_{fjm,r-1} Z_{fjmr} \le Q_{fjmr} \le \bar{Q}_{fjmr} Z_{fjmr}, \quad \forall\, f, j, m, r = 1, ..., \mathrm{NR}_{fjm} \tag{30}$$

$$\sum_{i \in I_f} Q_{ijm} = \sum_{r=1}^{\mathrm{NR}_{fjm}} Q_{fjmr}, \quad \forall\, f, j, m \tag{31}$$

$$C_{fjm} = \sum_{r=1}^{\mathrm{NR}} \left[ \bar{C}_{fjm,r-1} Z_{fjmr} + (Q_{fjmr} - \bar{Q}_{fjm,r-1} Z_{fjmr}) \frac{\bar{C}_{fjmr} - \bar{C}_{fjm,r-1}}{\bar{Q}_{fjmr} - \bar{Q}_{fjm,r-1}} \right], \quad \forall\, f, j, m \tag{32}$$

(ii) For the transportation of material between warehouses and distribution centers:

$$\sum_{r=1}^{\mathrm{NR}_{fmk}} Z_{fmkr} = 1, \quad \forall\, f, m, k \tag{33}$$

$$\bar{Q}_{fmk,r-1} Z_{fjmk} \le Q_{fmkr} \le \bar{Q}_{fmkr} Z_{fmkr}, \quad \forall\, f, m, k, r = 1, ..., \mathrm{NR}_{fmk} \tag{34}$$

$$\sum_{i \in I_f} Q_{imk} = \sum_{r=1}^{\mathrm{NR}_{fmk}} Q_{fmkr}, \quad \forall\, f, m, k \tag{35}$$

$$C_{fmk} = \sum_{r=1}^{\mathrm{NR}} \left[ \bar{C}_{fmk,r-1} Z_{fmkr} + (Q_{fmkr} - \bar{Q}_{fmk,r-1} Z_{fmkr}) \frac{\bar{C}_{fmkr} - \bar{C}_{fmk,r-1}}{\bar{Q}_{fmkr} - \bar{Q}_{fmk,r-1}} \right], \quad \forall\, f, m, k \tag{36}$$

(iii) For the transportation of material between distribution centers and customer zones:

$$\sum_{r=1}^{\mathrm{NR}_{fkl}} Z_{fklr} = 1, \quad \forall\, f, k, l \tag{37}$$

$$\bar{Q}_{fkl,r-1} Z_{fklr} \le Q_{fklr} \le \bar{Q}_{fklr} Z_{fklr}, \quad \forall\, f, k, l, r = 1, ..., \mathrm{NR}_{fkl} \tag{38}$$

$$\sum_{i \in I_f} Q_{ikl} = \sum_{r=1}^{\mathrm{NR}_{fkl}} Q_{fklr}, \quad \forall\, f, k, l \tag{39}$$

$$C_{jkl} = \sum_{r=1}^{\mathrm{NR}} \left[ \bar{C}_{fkl,r-1} Z_{fklr} + (Q_{fklr} - \bar{Q}_{fkl,r-1} Z_{fklr}) \frac{\bar{C}_{fklr} - \bar{C}_{fkl,r-1}}{\bar{Q}_{fklr} - \bar{Q}_{fkl,r-1}} \right], \quad \forall\, f, m, k \tag{40}$$

**3.3.5. Overall Objective Function.** By combining the cost terms derived in sections 3.3.1 and 3.3.4, we obtain the total cost of the supply chain network which is to be minimized by the optimization:

$$\min \sum_m C_m^{\mathrm{W}} Y_m + \sum_k C_k^{\mathrm{D}} Y_k + \sum_{i,j} C_{ij}^{\mathrm{P}} P_{ij} + \sum_{i,m} C_{\mathrm{im}}^{\mathrm{WH}} (\sum_j Q_{ijm}) + \sum_{i,k} C_{ik}^{\mathrm{DH}} (\sum_m Q_{imk}) + \sum_{f,j,m} C_{fjm} + \sum_{f,m,k} C_{fmk} + \sum_{f,k,l} C_{fkl} \tag{41}$$

The above minimization is subject to all of the constraints presented in section 3.2 as well as constraints (29)−(40).

**3.4. Calculation of Upper Bounds on Network Flows.** Constraints (6)−(8) involve the upper bounds $Q_{ijm}^{\max}$, $Q_{imk}^{\max}$, and $Q_{ikl}^{\max}$. The tightness of the MILP formulation and, consequently, the efficiency of its solution will depend crucially on the quality of these bounds.

To obtain estimates for these bounds, consider the flow of a product $i$ along the arc A → B connecting two nodes A and B in a network (see Figure 5). This flow, denoted by $Q_{i\mathrm{AB}}$, cannot exceed either the total flow of product $i$ entering node A or the total flow of product $i$ leaving node B. Thus, we obtain the expression

$$Q_{i\mathrm{AB}}^{\max} = \min\,(\sum_{\mathrm{C} \in \mathrm{IN}} Q_{i\mathrm{CA}}^{\max}, \sum_{\mathrm{C} \in \mathrm{OUT}} Q_{i\mathrm{BC}}^{\max}) \tag{42}$$

Applying expression (42) to the flows taking place in the supply chain network, we obtain Equation 43

$$Q_{ijm}^{\max} = \min\,(P_{ij}^{\max}, \sum_k Q_{imk}^{\max}) \quad \forall\, i, j, m \tag{43}$$

$$Q_{imk}^{\max} = \min\,(\sum_j Q_{ijm}^{\max}, \sum_l Q_{ikl}^{\max}) \quad \forall\, i, m, k \tag{44}$$

$$Q_{ikl}^{\max} = \min\,(\sum_m Q_{imk}^{\max}, D_{il}) \quad \forall\, i, k, l \tag{45}$$

considers the production rate $P_{ij}$ of product $i$ at plant $j$ as a flow notionally entering plant node $j$. Similarly eq 45 considers the demand rate $D_{il}$ for product $i$ at a customer zone $l$ as a fixed flow leaving customer zone node $l$.

The above formulas need to be applied in an iterative manner. Initially, we set $Q_{ijm}^{\max} = +\infty$, $Q_{imk}^{\max} = +\infty$, and $Q_{ikl}^{\max} = +\infty$. Then we repeatedly apply eqs 40−42 until none of the above upper bounds changes.

## 4. Supply Chain Network Design under Uncertainty in Product Demands

The formulation presented in section 3 assumes that the product demands, $D_{il}$, are constant and aims to

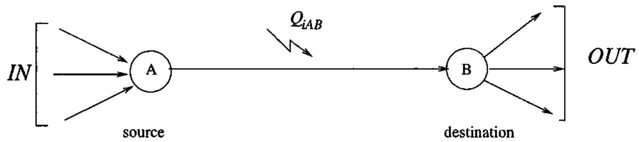


**Figure 5.** Obtaining upper bounds for network flows.

design a production and distribution network capable of handling them. We now proceed to consider the case where product demands are not known exactly but are subject to some uncertainty.

A good description of the uncertainties that occur throughout the entire supply chain network, affecting its performance and generally its operation, has been given by Davis.[29] He considers uncertainty arising from suppliers, manufacturing, and customers. Suppliers can be characterized through their past performance, and their responsiveness can be predicted with reasonable accuracy. Manufacturing problems can be addressed using reliability and maintenance analysis for the equipment. Finally, customer demands involve uncertainty which needs to be addressed via high quality forecasting methods.

At a more general level, Zimmermann[30] identifies the sources of uncertainty as lack of information, complexity of information, conflicting evidence, ambiguity, and measurement errors.

**4.1. Handling Uncertainty in Supply Chain Optimization.** Most of the factors affecting the operation of the supply chain network can be classified as either short-term fluctuations or long-term trends. To a certain extent, short-term fluctuations are captured implicitly in steady-state models such as the one presented in section 3 by averaging each flow in the network over a sufficiently long period of time. On the other hand, taking into account long-term variations necessitates a more direct approach.

Most research on addressing uncertainty can be distinguished as two primary approaches, referred as the *probabilistic approach* and the *scenario planning approach*. As argued by Zimmermann,[30] the choice of the appropriate method is context-dependent, with no single theory being sufficient to model all kinds of uncertainty.

Probabilistic models consider the uncertainty aspects of the supply chain treating one or more parameters as random variables with known probability distributions.[28] This approach has been adopted by Cohen and Lee,[31] Svoronos and Zipkin,[32] Lee and Billington,[33] Pyke and Cohen,[34] and Lee et al.[36]

On the other hand, scenario planning attempts to capture uncertainty by representing it in terms of a moderate number of discrete realizations of the stochastic quantities, constituting distinct scenarios.[37] Each complete realization of all uncertain parameters gives rise to a scenario.[28] The objective is to find robust solutions which perform well under *all* scenarios. In some applications, scenario planning replaces forecasting as a way of taking into account potential changes and trends in a business environment.

These are various common approaches to robust optimization[38] seeking, for example, to optimize the expected performance over all scenarios, to optimize the worst-case scenario, or to minimize the expected or worst-case "regret" across all scenarios.

Mulvey[39] uses scenario planning for formulating and solving operational problems, while Jenkins[40] employed this approach to assess the environmental impact of possible disasters.

Mohamed[41] uses a scenario approach to decide on the design of a production and distribution network that operates under varying exchange rates. A number of scenarios for different exchange rates aim to determine the production policy of the company, which operates in more than two countries. One important issue that arises in the context of the scenario planning approach is the increase in computational complexity as the number of scenarios increases (see, e.g., Cheung and Powell[42]). One approach toward addressing this concern is via the use of parallel computation.[43] Alternatively, specialized solution techniques have been considered (e.g., Ahmed and Sahinidis[44]). Very recently, Mir-Hassani et al.[45] have proposed a heuristic approach for handling very large numbers of scenarios.

**4.2. Scenario Generation.** In this paper, we adopt a scenario planning approach for handling the uncertainty in product demands. A question that needs to be addressed in this context concerns the generation of the scenarios to be considered. It is, of course, possible to assume that the demand for each product in each customer zone is an independent random parameter. However, more realistically, demands for similar products will tend to be correlated and will ultimately be controlled by a small number of major factors such as economic growth, political stability, competitor actions, and so on. This view is consistent with that of Mobasheri et al.,[46] who describe scenarios as plausible possible states derived from the present state with consideration of potential major industry events.

The overall aim should be to construct a set of scenarios representative of both optimistic and pessimistic situations within a risk analysis strategy. An example of such an approach is the Towers Perin software tool described by Mulvey.[40] A 12-step procedure for generating appropriate scenarios and a discussion on the use of scenario planning techniques are presented by Vanston et al.[47]

From the practical point of view, the main conclusion of the above discussion is that the total number of scenarios that have to be considered is typically much smaller than what might be expected given the (often large) numbers of products and customer zones. In any case, for the purposes of this paper, we will assume that there is some systematic way of generating product demand estimates $D_{il}^{[s]}$ for a number of scenarios $s = 1, ..., NS$.

**4.3. Mathematical Formulation.** The formulation of section 3 needs to be modified to take into account the multiple scenarios which are used to capture the uncertainty aspects (cf. section 4.1). Because we are still aiming at a *single* network design, the binary variables $Y_m$, $Y_k$, $X_{mk}$, and $X_{kl}$ and the capacity variables $W_k$ and $D_k$ remain unchanged. However, the operating variables relating to production and transportation flows will be different depending on which demand scenario materializes. Thus, we introduce a superscript $[s]$ on the corresponding variables which now become $P_{ij}^{[s]}$, $Q_{ijm}^{[s]}$, $Q_{imk}^{[s]}$, and $Q_{ikl}^{[s]}$. Any constraint that involves these variables must be enforced separately for each scenario. For example, constraint (7) now becomes

$$Q_{imk}^{[s]} \leq Q_{imk}^{[s],\max} X_{mk}, \quad \forall\ i, m, k, s = 1, ..., NS \quad (7')$$

The rest of the constraints in the formulation can be

**Table 2. Maximum Production Capacity of Each Plant for Each Product**

| plant | maximum production capacity (te/week) | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 | P9 | P10 | P11 | P12 | P13 | P14 |
| PL1 | 158 | 2268 | 1701 | 1512 | 0 | 812 | 642 | 482 | 320 | 504 | 0 | 661 | 441 | 221 |
| PL2 | 0 | 1411 | 1058 | 1328 | 996 | 664 | 664 | 0 | 0 | 0 | 530 | 496 | 330 | 0 |
| PL3 | 972 | 778 | 607 | 540 | 0 | 416 | 416 | 312 | 208 | 0 | 403 | 0 | 270 | 0 |

derived in a similar manner from those in section 3 and will not be repeated here. To arrive at a meaningful objective function for the optimization, we assume that the probability of scenario $s$ occurring in practice is known and is denoted by $\psi_s$. These probabilities will generally satisfy

$$\sum_{s=1}^{NS} \psi_s = 1 \quad (46)$$

Our aim is to minimize the *expected value* of the cost of the network taken over all of the scenarios. This leads to the modified objective function

$$\min \sum_m C^{W}_{m} Y_{m} + \sum_k C^{D}_{k} Y_{k} + \sum_{s=1}^{NS} \psi_s (\sum_{i,j} C^{P}_{ij} P^{[s]}_{ij} + \sum_{i,m} C^{WH}_{im} (\sum_j Q^{[s]}_{ijm}) + \sum_{i,k} C^{DH}_{ik} (\sum_m Q^{[s]}_{imk}) + \sum_{f,j,m} C^{[s]}_{fjm} + \sum_{f,k,l} C^{[s]}_{fmk} + \sum_{f,k,l} C^{[s]}_{fkl}) \quad (47)$$

**(a) Scenario-Dependent Distribution Network Structure.** The above discussion was based on the assumption that the structure of the distribution network (i.e., the transportation links between warehouses, distribution centers, and customer zones) was independent of the scenario. In many cases, this is unnecessary because the cost associated with the establishment of a transportation link is relatively small. This is especially the case when transportation is outsourced to third parties. In such cases, we could allow the network of transportation links to be different from one scenario to another—effectively postponing the decision until the actual demands are known.

The only change to the model formulation discussed earlier in this section is that now variables $X_{mk}$ and $X_{kl}$ also have a superscript $[s]$ to denote that they can take different values for each scenario. For example, constraint (7′) now becomes

$$Q^{[s]}_{imk} \le Q^{[s],\max}_{imk} X^{[s]}_{mk}, \quad \forall\ i, m, k, s = 1, ..., NS \quad (7'')$$

while the objective function (47) remains unchanged.

## 5. A Case Study in the Steady-State Design of Supply Chain Networks

To illustrate the applicability of the mathematical formulations presented in this paper, we consider three manufacturing plants producing 14 different types of products and located in three different European countries, namely, the United Kingdom, Spain, and Italy (see Figure 6). Each plant produces several products using a number of shared production resources. However, no single plant produces the entire range of products.

Product demands are such that Europe can be divided into 18 customer zones located in 16 different countries. We consider the establishment of enough distribution centers (DCs) to cover the whole market. The distribution centers can be located anywhere in 15 countries and are to be supplied by a number of warehouses, the location of which is to be decided among six candidate places.

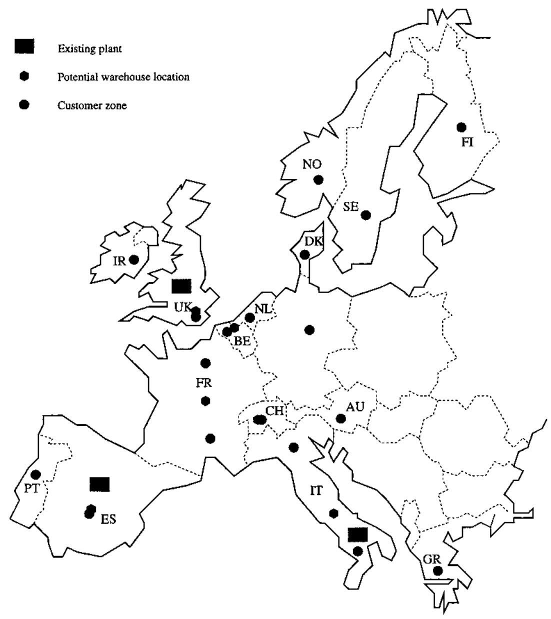


**Figure 6.** Europe-wide supply chain network design (candidate locations for distribution centers not shown).

**5.1. Problem Description. 5.1.1. Production Plants.** The maximum production capacity of each plant with respect to each product (i.e., the parameter $P^{\max}_{ij}$ of our formulation) is given in Table 2. The corresponding minimum production rates are taken to be zero (i.e., $P^{\min}_{ij} = 0,\ \forall\ i, j$).

The production of each product at each plant makes use of shared equipment resources as indicated in Table 3. The last column of this table lists the total rate of availability of each resource (in hours of useful operation per week). These are the parameters $R_{je}$ in constraint (16) of our formulation.

The unit production costs for the products are listed in Table 4.

**5.1.2. Warehouses and Distribution Centers.** The infrastructure costs for the establishment of the various warehouses and distribution centers under consideration are listed in Table 5; these have already been amortized and are expressed in £/week.

The warehouses and distribution centers are assumed to have maximum material handling capacities of 14 000 and 7000 te/week, respectively. All minimum handling capacities are set to zero. The coefficients $\alpha_{im}$ and $\beta_{ik}$ relating the capacity to the throughput of each material handled [cf. constraints (19) and (20)] are all taken to be unity. The unit handling costs (including labor or other operating costs) are also listed in Table 5; for the purposes of this case study, we assume that

**Table 3. Utilization and Availability Data for Shared Manufacturing Resources**

| plant resource | shared equipment resource utilization coefficient (h/te) | | | | | | | | | | | | | | total resource availability (h/week) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 | P9 | P10 | P11 | P12 | P13 | P14 | |
| PL1.E1 | | | | | | | | | | 0.2381 | | | | | 120 |
| PL1.E2 | | 0.0463 | 0.0617 | 0.0694 | | | | | | | | | | | 105 |
| PL1.E3 | | | | | | | 01.634 | 0.2178 | 0.3268 | | | | | | 105 |
| PL1.E4 | | | | | | | | | | | | 0.2267 | 0.3401 | 0.6802 | 150 |
| PL1.E5 | | | | | | 0.1292 | | | | | | | | | 105 |
| PL1.E6 | 0.6667 | | | | | | | | | | | | | | 105 |
| PL2.E1 | | | | | | | | | | | 0.1984 | 0.2118 | 0.3174 | | 105 |
| PL2.E2 | | | | 0.0793 | 0.1054 | 0.1582 | 0.1582 | | | | | | | | 105 |
| PL3.E3 | | 0.0740 | 0.1000 | | | | | | | | | | | | 120 |
| PL3.E2 | | | 0.1976 | 0.2222 | | | | | | | | | | | 165 |
| PL3.E3 | 0.1200 | 0.1543 | | | | 0.3968 | 0.3968 | 0.5291 | 0.7936 | | | | | | 120 |
| PL3.E4 | | | | | | | | | | | 0.2976 | | 0.4444 | | 120 |

**Table 4. Unit Production Costs**

| plant | unit production costs (£/te) | | | | | | |
|---|---|---|---|---|---|---|---|
| | P1 | P2 | P3 | P4 | P5 | P6 | P7 |
| PL1 | 61.27 | 61.27 | 61.27 | 61.27 | 61.27 | 61.27 | 256.90 |
| PL2 | 59.45 | 59.45 | 59.45 | 59.45 | 59.45 | 59.45 | 268.50 |
| PL3 | 61.44 | 61.44 | 61.44 | 61.44 | 61.44 | 61.44 | 270.80 |

| plant | unit production costs (£/te) | | | | | | |
|---|---|---|---|---|---|---|---|
| | P8 | P9 | P10 | P11 | P12 | P13 | P14 |
| PL1 | 256.90 | 256.90 | 61.27 | 256.90 | 256.90 | 256.90 | 256.90 |
| PL2 | 268.50 | 268.50 | 59.45 | 268.50 | 268.50 | 268.50 | 268.50 |
| PL3 | 270.80 | 270.80 | 61.44 | 270.80 | 270.80 | 270.80 | 270.80 |

**Table 5. Fixed Infrastructure and Material Handling Costs for Candidate Warehouses and Distribution Centers**

| warehouse/DC | infrastructure cost (£/week) | handling cost (£/te) |
|---|---|---|
| W1 | 10000 | 4.25 |
| W2 | 5000 | 4.55 |
| W3 | 4000 | 4.98 |
| W4 | 6000 | 4.93 |
| W5 | 6500 | 4.06 |
| W6 | 4000 | 5.28 |
| DC01 | 10000 | 4.25 |
| DC02 | 5000 | 4.55 |
| DC03 | 4000 | 4.98 |
| DC04 | 6000 | 4.93 |
| DC05 | 6500 | 4.85 |
| DC06 | 4000 | 3.90 |
| DC07 | 6000 | 4.06 |
| DC08 | 4000 | 3.08 |
| DC09 | 5000 | 6.00 |
| DC10 | 3000 | 4.85 |
| DC11 | 4500 | 4.12 |
| DC12 | 7000 | 5.66 |
| DC13 | 9000 | 5.28 |
| DC14 | 5500 | 4.95 |
| DC15 | 8500 | 4.83 |

**Table 6. Division of Products into Transportation Families**

| family | products | family | products |
|---|---|---|---|
| F1 | P1−P6, P10 | F3 | P11−P14 |
| F2 | P7−P9 | | |

they are the same for all products but may differ from one location to another.

**5.1.3. Transportation Costs.** The transportation costs are generally assumed to depend on the geographical distances between the locations involved. For the purposes of transportation, the 14 products may be aggregated into the three families shown in Table 6.

**Table 7. Baseline Unit Transportation Costs from Plants to Warehouses**

| plant | warehouses | | | | | |
|---|---|---|---|---|---|---|
| | W1 | W2 | W3 | W4 | W5 | W6 |
| | Family F1 Products (£/te) | | | | | |
| PL1 | 1.24 | 58.56 | 62.30 | 26.16 | 17.44 | 36.13 |
| PL2 | 60.82 | 1.68 | 70.96 | 43.93 | 70.96 | 55.76 |
| PL3 | 76.16 | 79.21 | 1.52 | 54.83 | 68.54 | 41.12 |
| | Family F2 Products (£/te) | | | | | |
| PL1 | 1.35 | 63.46 | 67.51 | 28.35 | 18.90 | 39.15 |
| PL2 | 82.70 | 2.29 | 96.48 | 59.72 | 96.48 | 75.81 |
| PL3 | 94.90 | 98.69 | 1.89 | 68.32 | 85.41 | 51.24 |
| | Family F3 Products (£/te) | | | | | |
| PL1 | 1.46 | 68.88 | 73.28 | 30.77 | 20.51 | 42.50 |
| PL2 | 79.69 | 2.21 | 92.97 | 57.55 | 92.97 | 73.05 |
| PL3 | 92.82 | 96.53 | 1.85 | 66.83 | 83.54 | 50.12 |

The baseline unit costs for transportation from plants to warehouses, warehouses to distribution centers, and distribution centers to customer zones are shown in Tables 7−9, respectively. These apply to transportation flows up to 40 te/week for each family. Above this limit, it is possible to deploy alternative models of transportation that have lower average unit costs. The magnitudes of these costs (relative to the corresponding baseline values) are shown in Table 10.

**5.2. Case Study 1: Deterministic Product Demands.** We start by considering a deterministic problem aiming to satisfy the demands shown in Table 11.

The optimal solution of the model of section 3 leads to the network structure shown in Figure 7. The number above each arc denotes the corresponding total material flow (in te/week), while the number next to each customer zone is the total demand at this zone (also in te/week). Tables 12−15 present the costs and the flows for each stage in the distribution chain.

The market is served by three warehouses and three distribution centers located close to the production sites. This is primarily a result of the demand patterns considered because the three countries which host the manufacturing plants are also the biggest customers.

The corresponding computational statistics for this problem are summarized in Table 16. The MILP problems were solved using the CPLEX v6.5[48] code embedded within *ooMILP* (Tsiakis et al.[49]). As can been seen from the results of Table 16, computational demands are relatively modest in this case despite the relatively high number of integer variables. This can be attributed primarily to the low integrality gap of this formulation.

**5.3. Case Study 2: Uncertain Product Demands.** We now consider a case with three possible product demand scenarios. The first one is the same as that used

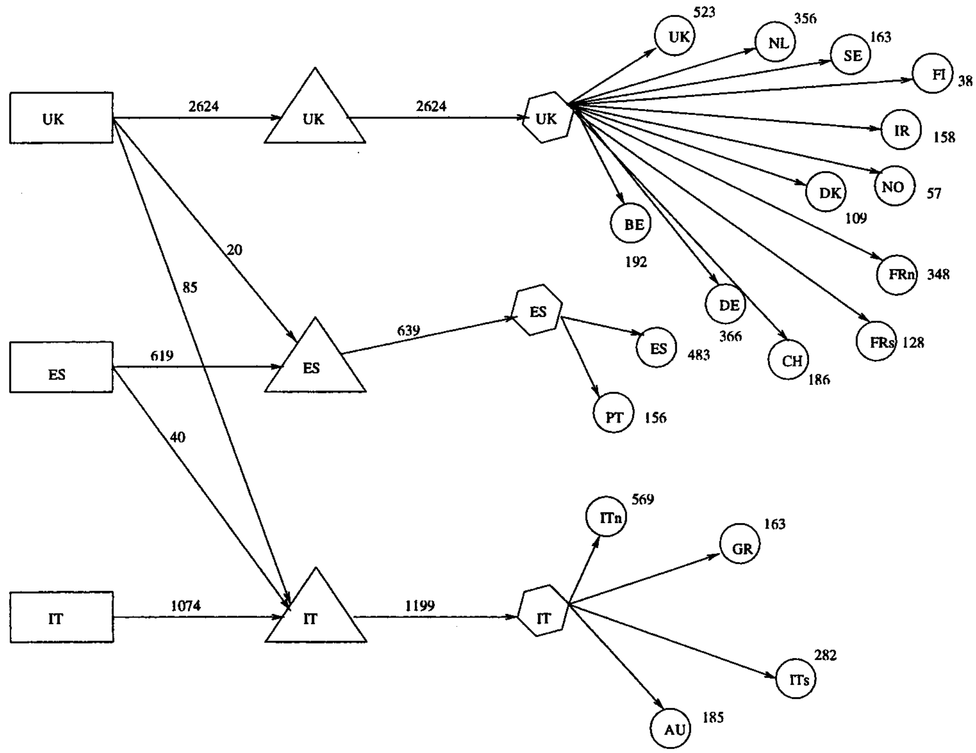


**Figure 7.** Optimal network for deterministic product demands.

**Table 8. Baseline Unit Transportation Costs from Warehouses to Distribution Centers**

| | distribution centers | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| warehouse | DC01 | DC02 | DC03 | DC04 | DC05 | DC06 | DC07 | DC08 | DC09 | DC10 | DC11 | DC12 | DC13 | DC14 | DC15 |
| | Family F1 Products (£/te) | | | | | | | | | | | | | | |
| W1 | 0 | 74.40 | 76.13 | 25.96 | 69.21 | 29.41 | 17.30 | 117.66 | 44.99 | 110.74 | 76.13 | 12.11 | 39.79 | 64.02 | 60.56 |
| W2 | 58.85 | 0 | 62.96 | 45.16 | 109.49 | 69.80 | 67.06 | 94.44 | 90.33 | 145.08 | 17.79 | 60.22 | 52.01 | 108.12 | 72.54 |
| W3 | 72.83 | 76.14 | 0 | 49.66 | 94.35 | 99.32 | 62.90 | 43.04 | 79.45 | 129.12 | 94.35 | 62.90 | 33.10 | 104.29 | 28.14 |
| W4 | 28.54 | 62.78 | 57.08 | 0 | 87.52 | 58.98 | 32.34 | 106.55 | 62.78 | 135.09 | 72.30 | 22.83 | 19.02 | 89.42 | 49.47 |
| W5 | 16.51 | 73.58 | 57.06 | 25.52 | 48.05 | 37.54 | 0 | 93.10 | 25.52 | 84.09 | 78.08 | 7.50 | 30.03 | 45.05 | 42.04 |
| W6 | 69.52 | 67.78 | 34.76 | 17.38 | 78.21 | 69.52 | 34.76 | 79.95 | 59.09 | 121.66 | 78.21 | 33.02 | 0 | 83.42 | 27.80 |
| | Family F2 Products (£/te) | | | | | | | | | | | | | | |
| W1 | 0 | 75.28 | 77.03 | 26.26 | 70.02 | 29.76 | 17.50 | 119.04 | 45.51 | 112.04 | 77.03 | 12.25 | 40.26 | 64.77 | 61.27 |
| W2 | 60.87 | 0 | 65.12 | 46.71 | 113.25 | 72.20 | 69.36 | 97.68 | 93.43 | 150.06 | 18.40 | 62.29 | 53.79 | 111.84 | 75.03 |
| W3 | 90.75 | 94.88 | 0 | 61.88 | 117.57 | 123.76 | 78.38 | 53.63 | 99.00 | 160.89 | 117.57 | 78.38 | 41.25 | 129.95 | 35.06 |
| W4 | 28.88 | 63.54 | 57.77 | 0 | 88.58 | 59.69 | 32.73 | 107.83 | 63.54 | 136.72 | 73.17 | 23.10 | 19.25 | 90.50 | 50.06 |
| W5 | 17.90 | 79.77 | 61.86 | 27.67 | 52.09 | 40.70 | 0 | 100.93 | 27.67 | 91.16 | 84.65 | 8.14 | 32.56 | 48.84 | 45.58 |
| W6 | 86.62 | 84.46 | 43.31 | 21.65 | 97.45 | 86.62 | 43.31 | 99.62 | 73.63 | 151.59 | 97.45 | 41.14 | 0 | 103.95 | 34.65 |
| | Family F3 Products (£/te) | | | | | | | | | | | | | | |
| W1 | 0 | 69.15 | 70.78 | 24.14 | 64.32 | 27.33 | 16.08 | 109.35 | 41.81 | 102.92 | 70.76 | 11.25 | 36.98 | 59.50 | 56.28 |
| W2 | 69.66 | 0 | 74.52 | 53.46 | 129.61 | 82.63 | 79.38 | 111.79 | 106.93 | 171.74 | 21.06 | 71.28 | 61.56 | 127.99 | 85.87 |
| W3 | 92.01 | 96.19 | 0 | 62.73 | 119.19 | 125.47 | 79.46 | 54.37 | 100.37 | 163.11 | 119.19 | 79.46 | 41.82 | 131.74 | 35.55 |
| W4 | 26.53 | 58.37 | 53.07 | 0 | 81.37 | 54.83 | 30.07 | 99.06 | 58.37 | 125.59 | 67.22 | 21.22 | 17.69 | 83.14 | 45.99 |
| W5 | 20.49 | 91.29 | 70.80 | 31.67 | 59.62 | 46.58 | 0 | 115.51 | 31.67 | 104.33 | 96.88 | 9.31 | 37.26 | 55.89 | 52.16 |
| W6 | 87.82 | 85.63 | 43.91 | 21.95 | 98.80 | 87.82 | 43.91 | 85.70 | 63.34 | 130.42 | 83.84 | 35.40 | 0 | 89.43 | 35.13 |

for the deterministic case considered in section 5.2. The demands for the second and third scenarios are given in Tables 17 and 18, respectively. We assume that all three scenarios are equally probable, i.e., $\psi_1 = \psi_2 = \psi_3 = {}^1/_3$. Our aim is to design a network that can handle all three scenarios while minimizing the expected cost.

To gain some understanding of the implications of the three demand patterns, we first consider each scenario in isolation by formulating and solving the corresponding deterministic design problem as described in section 3. The optimal network structures are shown in Figures 7−9, respectively. The three network structures differ in the allocation of products to plants, which depends strongly on the imposed demand patterns. More specifically, in the first scenario, the Italian network needs contributions from the U.K. and Spanish plants to cover the demands placed by its customers, while in the other two scenarios, it is capable of supplying the required amounts out of its own production. The set of customers assigned to each distribution center is also different in each scenario, which demonstrates the effects of transportation costs on the network structure.

We now consider all three scenarios simultaneously, making use of the scenario planning formulation of section 4. The optimal network structure obtained is shown in Figure 10. The three numbers above each arc denote the total flow rate (in te/week) of material over this link for the three scenarios considered. Similarly, the three numbers next to each customer zone are the corresponding total product demands.

**Table 9. Baseline Unit Transportation Costs from Distribution Centers to Customer Zones**

| | customer zones | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| DCs | CZ1 | CZ2 | CZ3 | CZ4 | CZ5 | CZ6 | CZ7 | CZ8 | CZ9 | CZ10 | CZ11 | CZ12 | CZ13 | CZ14 | CZ15 | CZ16 | CZ17 | CZ18 |
| Family F1 Products (£/te) | | | | | | | | | | | | | | | | | | |
| DC1 | 0 | 75.61 | 54.51 | 12.30 | 70.34 | 29.89 | 17.58 | 119.57 | 45.72 | 112.54 | 77.37 | 12.30 | 40.44 | 65.06 | 61.54 | 45.72 | 100.23 | 38.68 |
| DC2 | 73.55 | 0 | 78.68 | 73.55 | 136.84 | 87.23 | 83.81 | 118.02 | 112.89 | 181.31 | 22.23 | 75.26 | 64.99 | 135.12 | 90.65 | 34.21 | 71.84 | 94.07 |
| DC3 | 73.28 | 76.61 | 21.65 | 49.96 | 94.93 | 99.93 | 63.28 | 43.30 | 79.94 | 129.90 | 94.93 | 63.28 | 33.31 | 104.92 | 28.31 | 51.63 | 21.65 | 58.29 |
| DC4 | 26.58 | 58.47 | 53.16 | 14.17 | 81.51 | 54.93 | 30.12 | 99.23 | 58.47 | 125.81 | 67.33 | 21.26 | 17.72 | 83.28 | 46.07 | 54.93 | 17.72 | 40.75 |
| DC5 | 77.16 | 154.33 | 109.96 | 84.88 | 0 | 90.67 | 59.80 | 136.97 | 27.00 | 48.23 | 160.12 | 69.45 | 86.81 | 21.22 | 77.16 | 106.10 | 131.18 | 50.15 |
| DC6 | 27.08 | 84.65 | 79.57 | 38.93 | 79.57 | 0 | 42.32 | 143.90 | 57.56 | 118.51 | 77.87 | 38.93 | 67.72 | 67.72 | 88.03 | 67.72 | 126.97 | 64.33 |
| DC7 | 19.97 | 88.99 | 67.19 | 25.42 | 58.11 | 45.40 | 0 | 112.60 | 30.87 | 101.70 | 94.44 | 9.08 | 36.32 | 54.48 | 50.85 | 45.40 | 94.44 | 21.79 |
| DC8 | 118.02 | 121.49 | 64.22 | 109.34 | 123.23 | 147.53 | 107.61 | 0 | 116.29 | 149.27 | 142.32 | 109.34 | 79.84 | 137.12 | 60.74 | 86.78 | 27.77 | 93.72 |
| DC9 | 42.04 | 106.72 | 61.44 | 48.51 | 22.63 | 54.98 | 27.49 | 108.34 | 0 | 64.68 | 109.96 | 35.57 | 54.98 | 24.25 | 53.36 | 67.91 | 85.70 | 24.25 |
| DC10 | 107.82 | 178.57 | 116.24 | 117.92 | 42.11 | 117.92 | 94.34 | 144.88 | 67.38 | 0 | 181.94 | 104.45 | 117.92 | 50.54 | 104.45 | 134.77 | 144.88 | 84.23 |
| DC11 | 75.72 | 22.37 | 87.77 | 67.12 | 142.85 | 79.17 | 89.49 | 141.13 | 117.03 | 185.87 | 0 | 79.17 | 79.17 | 137.68 | 104.98 | 60.23 | 115.31 | 103.26 |
| DC12 | 11.91 | 74.91 | 44.26 | 13.62 | 61.29 | 39.15 | 8.51 | 107.26 | 37.45 | 105.56 | 78.31 | 0 | 30.64 | 61.29 | 49.37 | 39.15 | 88.53 | 25.53 |
| DC13 | 69.11 | 67.38 | 13.82 | 29.37 | 77.75 | 69.11 | 34.55 | 79.47 | 58.74 | 120.94 | 77.75 | 32.82 | 0 | 82.93 | 27.64 | 19.00 | 58.74 | 34.55 |
| DC14 | 63.10 | 134.74 | 88.69 | 71.63 | 18.76 | 68.22 | 51.16 | 134.74 | 25.58 | 51.16 | 136.44 | 61.40 | 81.86 | 0 | 80.16 | 95.51 | 127.92 | 51.16 |
| DC15 | 56.52 | 85.60 | 17.76 | 50.06 | 64.60 | 83.98 | 45.22 | 56.52 | 53.29 | 100.13 | 98.52 | 46.83 | 25.84 | 75.90 | 0 | 41.99 | 45.22 | 30.68 |
| Family F2 Products (£/te) | | | | | | | | | | | | | | | | | | |
| DC1 | 0 | 73.12 | 52.71 | 11.90 | 68.02 | 28.90 | 17.00 | 115.63 | 44.21 | 108.83 | 74.82 | 11.90 | 39.11 | 62.91 | 59.51 | 44.21 | 96.92 | 37.41 |
| DC2 | 73.20 | 0 | 78.31 | 73.20 | 136.20 | 86.82 | 83.42 | 117.47 | 112.36 | 180.46 | 22.13 | 74.91 | 64.69 | 134.49 | 90.23 | 34.05 | 71.50 | 93.63 |
| DC3 | 81.65 | 85.36 | 24.12 | 55.67 | 105.78 | 111.34 | 70.52 | 48.25 | 89.07 | 144.75 | 105.78 | 70.52 | 37.11 | 116.91 | 31.54 | 57.52 | 24.12 | 64.95 |
| DC4 | 24.76 | 54.48 | 49.53 | 13.20 | 75.95 | 51.18 | 28.06 | 92.46 | 54.48 | 117.22 | 62.74 | 19.81 | 16.51 | 77.60 | 42.92 | 51.18 | 16.51 | 37.97 |
| DC5 | 77.52 | 155.04 | 110.47 | 85.27 | 0 | 91.09 | 60.08 | 137.60 | 27.13 | 48.45 | 160.86 | 69.77 | 87.21 | 21.31 | 77.52 | 106.59 | 131.79 | 50.39 |
| DC6 | 32.65 | 102.06 | 95.93 | 46.94 | 95.93 | 0 | 51.03 | 173.50 | 69.40 | 142.88 | 93.89 | 46.94 | 81.64 | 81.64 | 106.14 | 81.64 | 153.09 | 77.56 |
| DC7 | 19.54 | 87.05 | 65.73 | 24.87 | 56.85 | 44.41 | 0 | 110.15 | 30.20 | 99.49 | 92.38 | 8.88 | 35.53 | 53.30 | 49.74 | 44.41 | 92.38 | 21.32 |
| DC8 | 127.18 | 130.92 | 69.20 | 117.83 | 132.79 | 158.98 | 115.96 | 0 | 125.31 | 160.85 | 153.37 | 117.83 | 86.03 | 147.76 | 65.46 | 93.52 | 29.92 | 101.00 |
| DC9 | 48.45 | 123.01 | 70.82 | 55.91 | 26.09 | 63.36 | 31.68 | 124.87 | 0 | 74.55 | 126.73 | 41.00 | 63.36 | 27.95 | 61.50 | 78.27 | 98.78 | 27.95 |
| DC10 | 111.91 | 185.36 | 120.66 | 122.40 | 43.71 | 122.40 | 97.92 | 150.38 | 69.94 | 0 | 188.85 | 108.41 | 122.40 | 52.46 | 108.41 | 139.89 | 150.38 | 87.43 |
| DC11 | 81.65 | 24.12 | 94.64 | 72.37 | 154.03 | 85.36 | 96.50 | 152.17 | 126.19 | 200.42 | 0 | 85.36 | 85.36 | 148.46 | 113.20 | 64.95 | 124.33 | 111.34 |
| DC12 | 12.44 | 78.21 | 46.21 | 14.22 | 63.99 | 40.88 | 8.88 | 111.98 | 39.10 | 110.21 | 81.76 | 0 | 31.99 | 63.99 | 51.55 | 40.88 | 92.43 | 26.66 |
| DC13 | 58.96 | 57.48 | 11.79 | 25.05 | 66.33 | 58.96 | 29.48 | 67.80 | 50.11 | 103.18 | 66.33 | 28.00 | 0 | 70.75 | 23.58 | 16.21 | 50.11 | 29.48 |
| DC14 | 60.11 | 128.35 | 84.48 | 68.23 | 17.87 | 64.98 | 48.74 | 128.35 | 24.37 | 48.74 | 129.97 | 58.48 | 77.98 | 0 | 76.36 | 90.98 | 121.85 | 48.74 |
| DC15 | 60.97 | 92.32 | 19.16 | 54.00 | 69.68 | 90.58 | 48.77 | 60.97 | 57.48 | 108.00 | 106.26 | 50.51 | 27.87 | 81.87 | 0 | 45.29 | 48.77 | 33.09 |
| Family F3 Products (£/te) | | | | | | | | | | | | | | | | | | |
| DC1 | 0 | 72.82 | 52.50 | 11.85 | 67.74 | 28.79 | 16.93 | 115.17 | 44.03 | 108.39 | 74.52 | 11.85 | 38.95 | 62.66 | 59.27 | 44.03 | 96.54 | 37.26 |
| DC2 | 69.04 | 0 | 73.86 | 69.04 | 128.46 | 81.89 | 78.68 | 110.80 | 105.98 | 170.21 | 20.87 | 70.65 | 61.02 | 126.85 | 85.10 | 32.11 | 67.44 | 88.31 |
| DC3 | 74.54 | 77.92 | 22.02 | 50.82 | 96.56 | 101.64 | 64.37 | 44.04 | 81.31 | 132.13 | 96.56 | 64.37 | 33.88 | 106.72 | 28.79 | 52.51 | 22.02 | 59.29 |
| DC4 | 28.74 | 63.22 | 57.48 | 15.32 | 88.13 | 59.39 | 32.57 | 107.29 | 63.22 | 136.03 | 72.80 | 22.99 | 19.16 | 90.05 | 49.81 | 59.39 | 19.16 | 44.06 |
| DC5 | 65.66 | 131.33 | 93.57 | 72.23 | 0 | 77.15 | 50.89 | 116.56 | 22.98 | 41.04 | 136.26 | 59.10 | 73.87 | 18.05 | 65.66 | 90.29 | 111.63 | 42.68 |
| DC6 | 31.07 | 97.12 | 91.29 | 44.67 | 91.29 | 0 | 48.56 | 165.10 | 66.04 | 135.96 | 89.35 | 44.67 | 77.69 | 77.69 | 101.00 | 77.69 | 145.68 | 73.81 |
| DC7 | 20.58 | 91.67 | 69.22 | 26.19 | 59.87 | 46.77 | 0 | 116.00 | 31.80 | 104.77 | 97.29 | 9.35 | 37.42 | 56.13 | 52.38 | 46.77 | 97.29 | 22.45 |
| DC8 | 121.40 | 124.97 | 66.05 | 112.48 | 126.76 | 151.75 | 110.69 | 0 | 119.62 | 153.54 | 146.40 | 112.48 | 82.12 | 141.04 | 62.48 | 89.27 | 28.56 | 96.41 |
| DC9 | 55.75 | 141.52 | 81.48 | 64.32 | 30.02 | 72.90 | 36.45 | 143.66 | 0 | 85.77 | 145.81 | 47.17 | 72.90 | 32.16 | 70.76 | 90.06 | 113.64 | 32.16 |
| DC10 | 106.08 | 175.70 | 114.37 | 116.03 | 41.44 | 116.03 | 92.82 | 142.55 | 66.30 | 0 | 179.02 | 102.77 | 116.03 | 49.72 | 102.77 | 132.60 | 142.55 | 82.88 |
| DC11 | 74.81 | 22.10 | 86.72 | 66.31 | 141.13 | 78.21 | 88.42 | 139.43 | 115.62 | 183.64 | 0 | 78.21 | 78.21 | 136.03 | 103.72 | 59.51 | 113.92 | 102.02 |
| DC12 | 13.58 | 85.41 | 50.47 | 15.53 | 69.88 | 44.64 | 9.70 | 122.30 | 42.70 | 120.36 | 89.29 | 0 | 34.94 | 69.88 | 56.29 | 44.64 | 100.94 | 29.11 |
| DC13 | 72.36 | 70.55 | 14.47 | 30.75 | 81.40 | 72.36 | 36.18 | 83.21 | 61.50 | 126.63 | 81.40 | 34.37 | 0 | 86.83 | 28.94 | 19.90 | 61.50 | 36.18 |
| DC14 | 72.66 | 155.15 | 102.12 | 82.48 | 21.60 | 78.56 | 58.92 | 155.15 | 29.46 | 58.92 | 157.12 | 70.70 | 94.27 | 0 | 92.30 | 109.98 | 147.30 | 58.92 |
| DC15 | 62.62 | 94.83 | 19.68 | 55.47 | 71.57 | 93.04 | 50.10 | 62.62 | 59.05 | 110.94 | 109.15 | 51.89 | 28.63 | 84.10 | 0 | 46.52 | 50.10 | 33.99 |

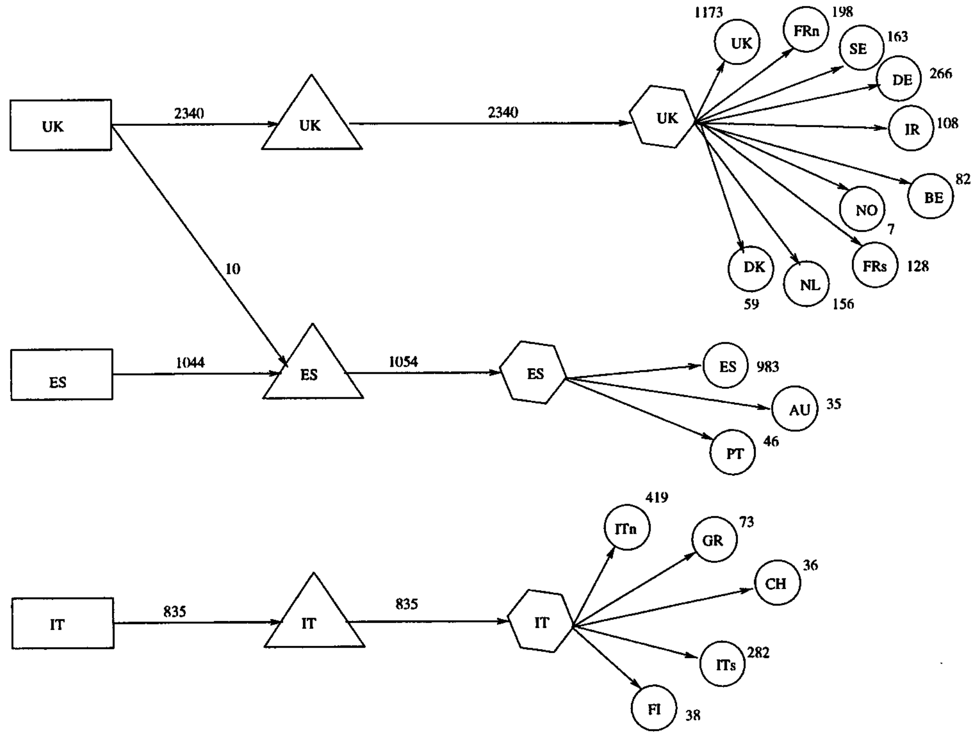


**Figure 8.** Optimal network structure for scenario 2.

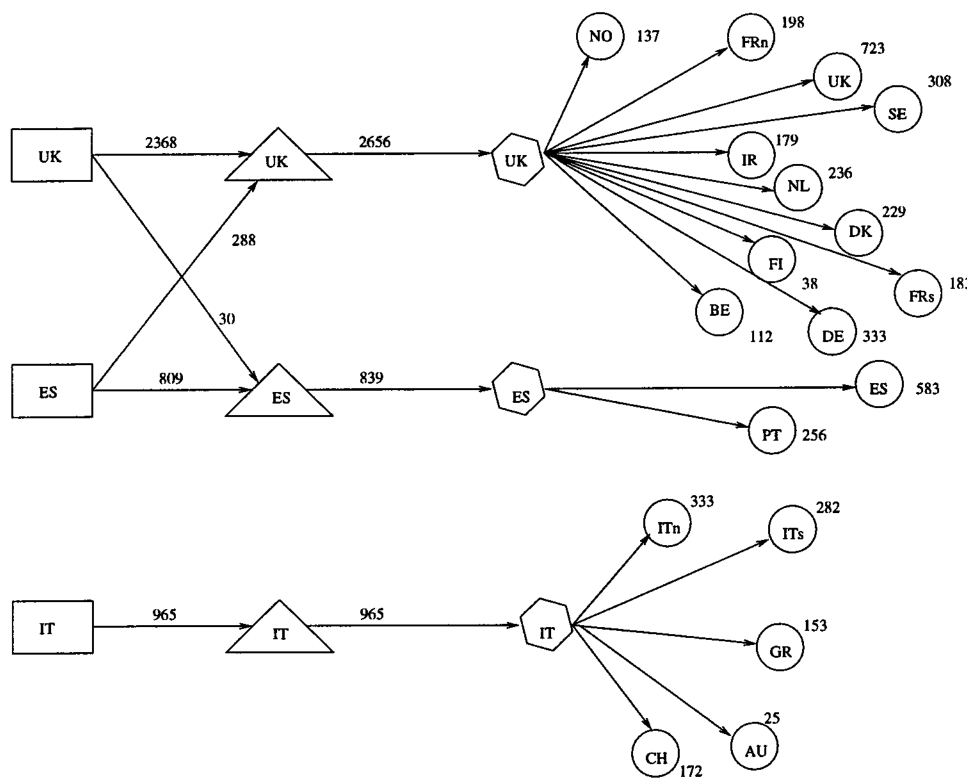


**Figure 9.** Optimal network structure for scenario 3.

**Table 10. Economies of Scale in Transportation Costs**

| amount transported (te/week) | transportation cost relative to the baseline |
|---|---|
| 0−40 | 1.00 |
| 40−100 | 0.95 |
| 100−1000 | 0.89 |
| 1000−5000 | 0.80 |

It is interesting to compare the expected costs of the network to the costs that would be incurred if one were to fix the structure of the network to those shown in Figures 7−9, respectively, optimizing the operational decisions only. As can be seen from Table 19, the fixed structure of Figure 7 has the same expected cost over the three demand scenarios as the optimal solution for the three scenarios case. This is due to the fact that the corresponding network configurations are the same (cf. Figures 7 and 10). On the other hand, the networks of Figures 8 and 9 result in more expensive operations than the optimal solution of Figure 10.

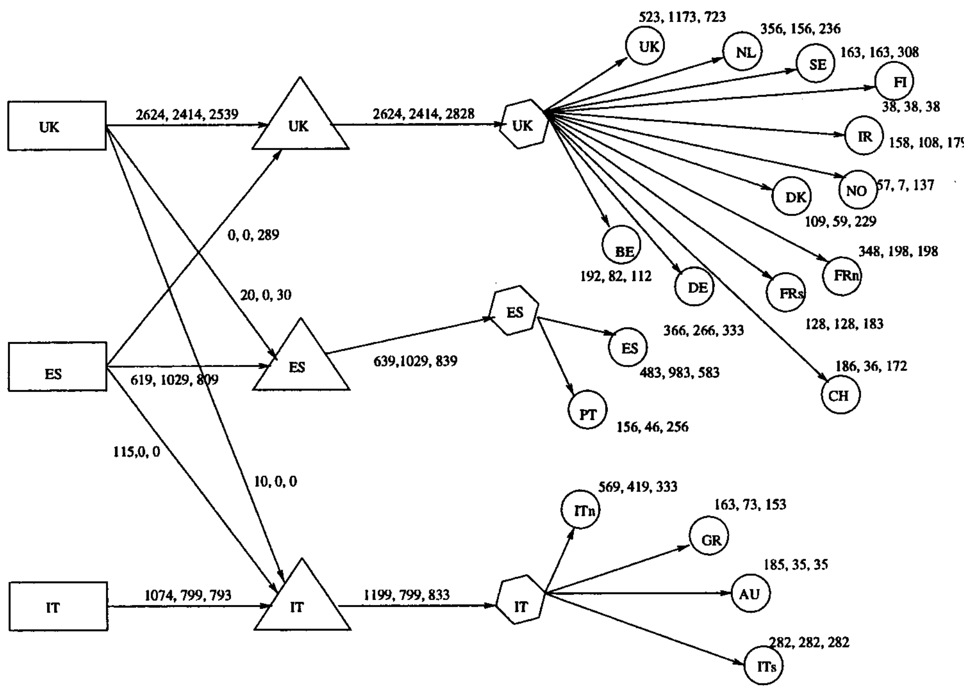


**Figure 10.** Optimal network structure for all three scenarios considered simultaneously.

**Table 11. Product Demands by Customer Zone**

| | product demands (te/week) | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| customer zone | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 | P9 | P10 | P11 | P12 | P13 | P14 |
| CZ01 | 18 | 0 | 0 | 106 | 0 | 252 | 0 | 0 | 43 | 0 | 0 | 0 | 70 | 34 |
| CZ02 | 0 | 99 | 55 | 203 | 76 | 0 | 30 | 0 | 0 | 0 | 20 | 0 | 0 | 0 |
| CZ03 | 0 | 155 | 50 | 266 | 0 | 66 | 17 | 0 | 0 | 0 | 0 | 0 | 15 | 0 |
| CZ04 | 15 | 150 | 126 | 0 | 0 | 0 | 5 | 27 | 0 | 0 | 0 | 25 | 0 | 0 |
| CZ05 | 0 | 0 | 92 | 0 | 0 | 0 | 0 | 0 | 21 | 0 | 0 | 0 | 50 | 0 |
| CZ06 | 0 | 0 | 50 | 0 | 0 | 68 | 0 | 0 | 20 | 0 | 0 | 0 | 10 | 10 |
| CZ07 | 0 | 114 | 0 | 140 | 0 | 0 | 0 | 0 | 34 | 0 | 0 | 0 | 68 | 0 |
| CZ08 | 0 | 50 | 0 | 45 | 40 | 23 | 0 | 0 | 5 | 0 | 0 | 0 | 0 | 0 |
| CZ09 | 0 | 50 | 0 | 0 | 0 | 0 | 52 | 0 | 7 | 0 | 0 | 0 | 0 | 0 |
| CZ10 | 0 | 0 | 0 | 17 | 0 | 0 | 0 | 0 | 5 | 0 | 0 | 0 | 16 | 0 |
| CZ11 | 0 | 50 | 0 | 31 | 20 | 0 | 0 | 0 | 0 | 0 | 20 | 0 | 15 | 20 |
| CZ12 | 10 | 31 | 0 | 0 | 0 | 100 | 13 | 0 | 0 | 38 | 0 | 0 | 0 | 0 |
| CZ13 | 0 | 21 | 0 | 100 | 0 | 0 | 15 | 0 | 0 | 0 | 0 | 0 | 50 | 0 |
| CZ14 | 0 | 0 | 0 | 50 | 0 | 0 | 0 | 7 | 0 | 0 | 0 | 0 | 0 | 0 |
| CZ15 | 0 | 0 | 0 | 150 | 0 | 0 | 10 | 0 | 0 | 0 | 15 | 0 | 0 | 10 |
| CZ16 | 15 | 0 | 68 | 0 | 0 | 0 | 5 | 20 | 0 | 0 | 0 | 20 | 0 | 0 |
| CZ17 | 0 | 103 | 0 | 110 | 0 | 44 | 12 | 0 | 0 | 0 | 0 | 0 | 13 | 0 |
| CZ18 | 0 | 0 | 0 | 0 | 0 | 100 | 0 | 0 | 0 | 266 | 0 | 0 | 0 | 0 |

**Table 12. Costs Associated with Manufacturing Plants**

| plant | production (te/week) | manufacturing cost (£/week) |
|---|---|---|
| U.K. | 2729 | 295 516 |
| ES | 659 | 57 201 |
| IT | 1074 | 84 175 |
| total | 4462 | 436 892 |

**Table 13. Costs Associated with Warehouses**

| warehouse | infrastructure cost (£/week) | throughput (te/week) | handling cost (£/week) |
|---|---|---|---|
| U.K. | 10 000 | 2624 | 11 152 |
| ES | 5 000 | 639 | 2 907 |
| IT | 4 000 | 1199 | 5 971 |
| total | 19 000 | 4462 | 20 030 |

Tables 20–23 present the costs and flows for each stage in the distribution chain for each scenario in the optimal solution of the scenario planning formulation.

Some computational results for this case are shown in Table 24. A comparison with the corresponding statistics for the deterministic design problem (cf. Table 16) indicates that both the integrality gap of the formulation and the performance of the branch-and-bound algorithm remain reasonable (e.g., with respect to the number of nodes examined). However, the overall computational cost grows significantly because of the increase in the number of constraints and variables.

**Table 14. Costs Associated with Distribution Centers**

| distribution center | infrastructure cost (£/week) | throughput (te/week) | handling cost (£/week) |
|---|---|---|---|
| U.K. | 10 000 | 2624 | 11 152 |
| ES | 5 000 | 639 | 2 907 |
| IT | 4 000 | 1199 | 5 971 |
| total | 19 000 | 4462 | 20 030 |

## 6. Conclusions

The design of supply chain networks is a difficult task because of the intrinsic complexity of the major subsystems of these networks and the many interactions

**Table 15. Transportation Costs throughout the System**

| plant location | warehouse location | warehouse flow (te/week) | warehouse cost[a] (£/week) | distribution center location | distribution center flow (te/week) | distribution center cost[a] (£/week) | customer zone location | customer zone flow (te/week) | customer zone cost[a] (£/week) |
|---|---|---|---|---|---|---|---|---|---|
| U.K. | U.K. | 2634 | 3936 | U.K. | 2634 | 0.0 | U.K. | 523 | 1167 |
| | | | | | | | FRn | 198 | 5484 |
| | | | | | | | SE | 163 | 16152 |
| | | | | | | | IR | 158 | 6524 |
| | | | | | | | NL | 356 | 8136 |
| | | | | | | | DK | 109 | 7108 |
| | | | | | | | FI | 38 | 6267 |
| | | | | | | | NO | 7 | 5408 |
| | | | | | | | BE | 192 | 3086 |
| | | | | | | | CH | 186 | 10360 |
| | | | | | | | FRs | 128 | 8314 |
| | | | | | | | DE | 366 | 17451 |
| | ES | 20 | 2066 | | | | | | |
| | IT | 85 | 7778 | | | | | | |
| ES | ES | 619 | 1380 | ES | 639 | 0.0 | ES | 483 | 1034 |
| | | | | | | | PT | 156 | 4802 |
| | IT | 40 | 4258 | | | | | | |
| IT | IT | 1074 | 2046 | IT | 1199 | 0.0 | ITn | 569 | 15235 |
| | | | | | | | GR | 163 | 9351 |
| | | | | | | | AU | 185 | 7161 |
| | | | | | | | ITs | 282 | 7872 |
| Total | | 4462 | 21464 | | 4462 | 10769 | | 4462 | 141002 |

[a] This is the cost incurred for material to be transported *to* this destination.

**Table 16. Computational Statistics for the Deterministic Design Problem**

| | | | |
|---|---|---|---|
| no. of constraints | 17949 | integrality gap | 0.00% |
| no. of integer variables | 4917 | optimality margin | 0.1% |
| no. of continuous variables | 11025 | no. of branch-and-bound nodes | 2634 |
| optimal objective value | 677458 | CPU time (SUN UltraSPARC60) | 1082 |
| fully relaxed LP objective function | 677458 | solver | CPLEX6.5 |

**Table 17. Product Demands by Customer Zone for Scenario 2**

| customer zone | product demands (te/week) P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 | P9 | P10 | P11 | P12 | P13 | P14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CZ01 | 18 | 0 | 0 | 506 | 0 | 452 | 0 | 0 | 43 | 0 | 0 | 0 | 120 | 34 |
| CZ02 | 0 | 499 | 155 | 203 | 76 | 0 | 30 | 0 | 0 | 0 | 20 | 0 | 0 | 0 |
| CZ03 | 0 | 155 | 0 | 166 | 0 | 66 | 17 | 0 | 0 | 0 | 0 | 0 | 15 | 0 |
| CZ04 | 15 | 0 | 126 | 0 | 0 | 0 | 5 | 27 | 0 | 0 | 0 | 25 | 0 | 0 |
| CZ05 | 0 | 0 | 92 | 0 | 0 | 0 | 0 | 0 | 21 | 0 | 0 | 0 | 50 | 0 |
| CZ06 | 0 | 0 | 0 | 0 | 0 | 68 | 0 | 0 | 20 | 0 | 0 | 0 | 10 | 10 |
| CZ07 | 0 | 14 | 0 | 40 | 0 | 0 | 0 | 0 | 34 | 0 | 0 | 0 | 68 | 0 |
| CZ08 | 0 | 0 | 0 | 45 | 0 | 23 | 0 | 0 | 5 | 0 | 0 | 0 | 0 | 0 |
| CZ09 | 0 | 0 | 0 | 0 | 0 | 0 | 52 | 0 | 7 | 0 | 0 | 0 | 0 | 0 |
| CZ10 | 0 | 0 | 0 | 17 | 0 | 0 | 0 | 0 | 5 | 0 | 0 | 0 | 16 | 0 |
| CZ11 | 0 | 0 | 0 | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 15 | 0 |
| CZ12 | 0 | 31 | 0 | 0 | 0 | 0 | 13 | 0 | 0 | 38 | 0 | 0 | 0 | 0 |
| CZ13 | 0 | 21 | 0 | 0 | 0 | 0 | 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| CZ14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 0 | 0 | 0 | 0 | 0 | 0 |
| CZ15 | 0 | 0 | 0 | 0 | 0 | 0 | 10 | 0 | 0 | 0 | 15 | 0 | 0 | 0 |
| CZ16 | 15 | 0 | 68 | 0 | 0 | 0 | 5 | 20 | 0 | 0 | 0 | 20 | 0 | 0 |
| CZ17 | 0 | 103 | 0 | 110 | 0 | 44 | 12 | 0 | 0 | 0 | 0 | 0 | 13 | 0 |
| CZ18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 266 | 0 | 0 | 0 | 0 |

among these subsystems, as well as external factors such as the considerable uncertainty in product demands. In the past, this complexity has forced much of the research in this area to focus on individual components of supply chain networks. Recently, however, attention has increasingly been placed on the performance, design, and analysis of the supply chain as a whole.

This paper has proposed a model based on a detailed mathematical programming formulation that aims to address some of the complexity of the above problem. In particular, it considers flexible production facilities in which a number of products are produced, making use of shared resources. It also takes into account flexible transportation modes with economy-of-scale effects. Although the handling of uncertainty is demonstrated by considering uncertainties in product demands, other uncertainties (e.g., in unit production and/or transportation costs) can, in principle, be handled in a similar manner. Overall, it is hoped that this approach can lead to a quantitative tool to support strategic planning decisions in supply chain network design.

As is often the case with MILP-based formulations, one important issue is that of computational complexity, especially in the context of scenario planning approaches for the handling of uncertainty. Clearly, the identifica-

**Table 18. Product Demands by Customer Zone for Scenario 3**

| customer zone | product demands (te/week) P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 | P9 | P10 | P11 | P12 | P13 | P14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CZ01 | 38 | 0 | 0 | 206 | 0 | 252 | 0 | 0 | 43 | 30 | 0 | 0 | 120 | 34 |
| CZ02 | 0 | 199 | 155 | 103 | 76 | 0 | 30 | 0 | 0 | 0 | 20 | 0 | 0 | 0 |
| CZ03 | 0 | 155 | 0 | 80 | 0 | 66 | 17 | 0 | 0 | 0 | 0 | 0 | 15 | 0 |
| CZ04 | 15 | 0 | 126 | 0 | 0 | 0 | 5 | 27 | 0 | 0 | 0 | 25 | 0 | 0 |
| CZ05 | 0 | 0 | 92 | 100 | 0 | 0 | 0 | 0 | 21 | 0 | 45 | 0 | 50 | 0 |
| CZ06 | 0 | 0 | 0 | 0 | 36 | 68 | 15 | 0 | 20 | 30 | 0 | 0 | 10 | 10 |
| CZ07 | 0 | 14 | 0 | 80 | 0 | 0 | 0 | 40 | 34 | 0 | 0 | 0 | 68 | 0 |
| CZ08 | 0 | 0 | 0 | 45 | 0 | 23 | 0 | 0 | 5 | 0 | 40 | 0 | 0 | 0 |
| CZ09 | 0 | 150 | 0 | 0 | 0 | 0 | 52 | 20 | 7 | 0 | 0 | 0 | 0 | 0 |
| CZ10 | 0 | 0 | 0 | 17 | 0 | 0 | 0 | 0 | 5 | 0 | 0 | 0 | 16 | 0 |
| CZ11 | 0 | 150 | 0 | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 30 | 0 | 15 | 30 |
| CZ12 | 0 | 31 | 0 | 0 | 30 | 0 | 13 | 0 | 0 | 38 | 0 | 0 | 0 | 0 |
| CZ13 | 0 | 21 | 0 | 86 | 0 | 50 | 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| CZ14 | 0 | 0 | 0 | 120 | 0 | 0 | 0 | 7 | 20 | 0 | 0 | 0 | 0 | 0 |
| CZ15 | 0 | 0 | 0 | 0 | 0 | 0 | 10 | 0 | 0 | 0 | 15 | 0 | 0 | 0 |
| CZ16 | 15 | 0 | 68 | 0 | 0 | 0 | 5 | 20 | 0 | 0 | 0 | 20 | 0 | 55 |
| CZ17 | 0 | 103 | 0 | 110 | 0 | 44 | 12 | 0 | 0 | 0 | 0 | 0 | 13 | 0 |
| CZ18 | 0 | 0 | 0 | 0 | 17 | 0 | 0 | 0 | 0 | 266 | 0 | 0 | 50 | 0 |

**Table 19. Comparison of Network Structures**

| network structure | minimum expected cost over three demand scenarios (£/week) | difference from optimal (£/week) |
|---|---|---|
| Figure 7 | 1 957 046 | 0 |
| Figure 8 | 1 977 571 | 20 525 |
| Figure 9 | 1 962 922 | 5876 |
| Figure 10 | 1 957 046 | 0 |

**Table 20. Costs Associated with Manufacturing Plants**

| plant | production (te/week) | | | manufacturing cost (£) | | |
|---|---|---|---|---|---|---|
| | scenario 1 | scenario 2 | scenario 3 | scenario 1 | scenario 2 | scenario 3 |
| U.K. | 2654 | 2378 | 2569 | 290 896 | 270 566 | 285 749 |
| ES | 734 | 1029 | 1099 | 61 684 | 74 762 | 128 186 |
| IT | 1074 | 835 | 763 | 84 175 | 67 304 | 83 685 |
| total | 4462 | 4242 | 4460 | 436 755 | 412 632 | 497 620 |
| expected value | 4383 | | | 448 555 | | |

**Table 21. Costs Associated with Warehouses**

| warehouse W | infrastructure cost (£/week) | throughput (te/week) | | | handling cost (£/week) | | |
|---|---|---|---|---|---|---|---|
| | | scenario 1 | scenario 2 | scenario 3 | scenario 1 | scenario 2 | scenario 3 |
| U.K. | 10 000 | 2624 | 2414 | 2828 | 11 152 | 10 983 | 12 019 |
| ES | 5 000 | 639 | 1029 | 829 | 2 907 | 4 681 | 3 817 |
| IT | 4 000 | 1199 | 799 | 793 | 5 971 | 3 955 | 4 073 |
| total | 19 000 | 4462 | 4242 | 4460 | 20 030 | 19 619 | 18 680 |
| expected value | 19 000 | 4383 | | | 19 423 | | |

**Table 22. Costs Associated with Distribution Centers**

| distribution center | infrastructure cost (£/week) | throughput (te/week) | | | handling cost (£/week) | | |
|---|---|---|---|---|---|---|---|
| | | scenario 1 | scenario 2 | scenario 3 | scenario 1 | scenario 2 | scenario 3 |
| U.K. | 10 000 | 2624 | 2414 | 2828 | 11 152 | 10 983 | 12 019 |
| ES | 5 000 | 639 | 1029 | 829 | 2 907 | 4 681 | 3 817 |
| IT | 4 000 | 1199 | 799 | 793 | 5 971 | 3 955 | 4 073 |
| total | 19 000 | 4462 | 4242 | 4460 | 20 030 | 19 619 | 18 680 |
| expected value | 19 000 | 4383 | | | 19 423 | | |

tion of the true underlying sources of such uncertainty (e.g., major political and economic events) is key in generating a representative but not excessive number of scenarios.

Complementary to the derivation of the minimum possible number of scenarios is the use of special decomposition techniques that exploit the special structure of the "multiperiod" problem occurring in scenario-based optimization (see, for instance, Varvarezos and Grossmann[50]). However, a complication that arises in the case of the problem considered in this paper is the existence of discrete operational decisions (e.g., with respect to transportation) within each of the periods.[51] This reduces the applicability of decomposition techniques that rely on the availability of gradient information from the subproblems describing each period.[52]

**Table 23. Transportation Costs throughout the System**

| plant location | warehouse location | warehouse flow (te/week) Sc 1 | warehouse flow (te/week) Sc 2 | warehouse flow (te/week) Sc 3 | warehouse cost[a] (£/week) Sc 1 | warehouse cost[a] (£/week) Sc 2 | warehouse cost[a] (£/week) Sc 3 | distribution center location | distribution center flow (te/week) Sc 1 | distribution center flow (te/week) Sc 2 | distribution center flow (te/week) Sc 3 | distribution center cost[a] (£/week) Sc 1 | distribution center cost[a] (£/week) Sc 2 | distribution center cost[a] (£/week) Sc 3 | customer zone location | customer zone flow (te/week) Sc 1 | customer zone flow (te/week) Sc 2 | customer zone flow (te/week) Sc 3 | customer zone cost[a] (£/week) Sc 1 | customer zone cost[a] (£/week) Sc 2 | customer zone cost[a] (£/week) Sc 3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| U.K. | U.K. | 2624 | 2378 | 2539 | 3936 | 3657 | 3816 | U.K. | 2624 | 2378 | 2828 | 0.0 | 0.0 | 0.0 | U.K. | 523 | 1173 | 723 | 1167 | 2503 | 1575 |
| | | | | | | | | | | | | | | | FRn | 348 | 198 | 198 | 5484 | 3822 | 3822 |
| | | | | | | | | | | | | | | | SE | 163 | 163 | 308 | 16152 | 16152 | 28540 |
| | | | | | | | | | | | | | | | IR | 158 | 108 | 179 | 6524 | 4624 | 7735 |
| | | | | | | | | | | | | | | | NL | 356 | 156 | 236 | 8136 | 3888 | 4821 |
| | | | | | | | | | | | | | | | DK | 109 | 59 | 229 | 7108 | 3771 | 12821 |
| | | | | | | | | | | | | | | | FI | 38 | 38 | 38 | 6267 | 6267 | 6267 |
| | | | | | | | | | | | | | | | NO | 57 | 7 | 137 | 5408 | 660 | 13137 |
| | | | | | | | | | | | | | | | BE | 192 | 82 | 112 | 3086 | 1440 | 1930 |
| | | | | | | | | | | | | | | | CH | 186 | 36 | 172 | 10360 | 2147 | 9216 |
| | | | | | | | | | | | | | | | FRs | 128 | 128 | 183 | 8314 | 8314 | 11682 |
| | | | | | | | | | | | | | | | DE | 366 | 266 | 333 | 17541 | 12917 | 16414 |
| | ES | 20 | | 30 | 2066 | | 3099 | | | | | | | | | | | | | | |
| | IT | 10 | | | 1099 | | | | | | | | | | | | | | | | |
| ES | ES | 619 | 1029 | 809 | 1380 | 2175 | 1768 | ES | 639 | 1029 | 839 | 0.0 | 0.0 | 0.0 | ES | 483 | 983 | 583 | 1034 | 2041 | 1235 |
| | | | | | | | | | | | | | | | PT | 156 | 46 | 256 | 4802 | 1501 | 7451 |
| | U.K. | | | 289 | | | 28238 | | | | | | | | | | | | | | |
| | IT | 115 | | | 11223 | | | | | | | | | | | | | | | | |
| IT | IT | 1074 | 809 | 793 | 2046 | 1554 | 1600 | IT | 1199 | 809 | 823 | 0.0 | 0.0 | 0.0 | GR | 163 | 73 | 153 | 9351 | 4561 | 10044 |
| | | | | | | | | | | | | | | | ITn | 569 | 419 | 333 | 15235 | 11423 | 9237 |
| | | | | | | | | | | | | | | | AU | 185 | 35 | 35 | 7161 | 1118 | 1118 |
| | | | | | | | | | | | | | | | ITs | 282 | 282 | 282 | 7872 | 7872 | 7872 |
| total | | 4462 | 4242 | 4460 | 21464 | 7386 | 38521 | | 4462 | 4242 | 4460 | 0.0 | 0.0 | 0.0 | | 4462 | 4242 | 4460 | 141002 | 92176 | 151532 |
| expected value | | | 4383 | | | 22434 | | | | 4383 | | | 0.0 | | | | 4383 | | | 128108 | |

[a] This is the cost incurred for material to be transported *to* this destination.

**Table 24. Computational Statistics for the Scenario-Based Design Problem**

| | |
|---|---|
| no. of constraints | 54 228 |
| no. of integer variables | 13 989 |
| no. of continuous variables | 34 176 |
| optimal objective value | 1 957 046 |
| fully relaxed LP objective function | 1 955 740 |
| integrality gap | 0.07% |
| optimality margin | 0.1% |
| no. of branch-and-bound nodes | 3663 |
| CPU time (SUN UltraSPARC60) | 3855 |
| solver | CPLEX6.5 |

## Notation

$\bar{C}_{fjmr}$ = transportation cost for products of family $f$ from plant $j$ to warehouse $m$ over range $r$ of transportation flow

$\bar{C}_{fkmr}$ = transportation cost for products of family $f$ from warehouse $m$ to distribution center $k$ over range $r$ of transportation flow

$\bar{C}_{fklr}$ = transportation cost for products of family $f$ from distribution center $k$ to customer zone $l$ over range $r$ of transportation flow

$C_{im}^{WH}$ = unit handling cost for product $i$ at warehouse $m$

$C_{ik}^{DH}$ = unit handling cost for product $i$ at distribution center $k$

$C_m^{W}$ = annualized fixed cost of establishing a warehouse at location $m$

$C_k^{D}$ = annualized fixed cost of establishing a distribution center at location $k$

$C_{ij}^{P}$ = unit production cost for product $i$ at plant $j$

$\bar{C}_r$ = transportation cost which corresponds to interval $r$

$C_{ijm}$ = actual transportation cost of product $i$ from plant $j$ to warehouse $m$

$C_{ikm}$ = actual transportation cost of product $i$ from warehouse $m$ to distribution center $k$

$C_{ikl}$ = actual transportation cost of product $i$ from distribution center $k$ to customer zone $l$

$D_k^{max}$ = maximum distribution center capacity

$D_k^{min}$ = minimum distribution center capacity

$D_{il}$ = demand for product $i$ in customer zone $l$

$D_k$ = capacity of distribution center $k$

$e$ = manufacturing resource (equipment, manpower, utilities, etc.)

$f$ = transportation product families

$i$ = products

$j$ = plants

$K^{ss}$ = set of distribution centers requiring single sourcing

$k$ = possible distribution centers

$L^{ss}$ = set of customer zones requiring single sourcing

$l$ = customer zones

$m$ = possible warehouses

NI = number of products

NF = number of families

$NR_{fjm}$ = number of transportation flow ranges, each with a constant unit cost for transporting product family $f$ from plant $j$ to warehouse $m$

$NR_{fmk}$ = number of transportation flow ranges, each with a constant unit cost for transporting product family $f$ from warehouse $m$ to distribution center $k$

$NR_{fkl}$ = number of transportation flow ranges, each with a constant unit cost for transporting product family $f$ from distribution center $k$ to customer $l$

NS = number of product demand scenarios

$P_{ij}^{max}$ = maximum production capacity of plant $j$ for product $i$

$P_{ij}^{min}$ = minimum production capacity of plant $j$ for product $i$

$P_{ij}$ = production rate of product $i$ in plant $j$

$Q_{jm}^{\min}$ = minimum rate of flow of material that can be practically and economically transferred from plant $j$ to warehouse $m$

$Q_{mk}^{\min}$ = minimum rate of flow of material that can be practically and economically transferred from warehouse $m$ to distribution center $k$

$Q_{kl}^{\min}$ = minimum rate of flow of material that can be practically and economically transferred from distribution center $k$ to customer zone $l$

$Q_{ijm}^{\max}$ = maximum rate of flow of product $i$ transferred from plant $j$ to warehouse $m$

$Q_{imk}^{\max}$ = maximum rate of flow of product $i$ transferred from warehouse $m$ to distribution center $k$

$Q_{ikl}^{\max}$ = maximum rate of flow of product $i$ transferred from distribution center $k$ to customer zone $l$

$\bar{Q}_{fjmr}$ = upper bound for range $r$ of transportation flow of product family $f$ from plant $j$ to warehouse $m$

$\bar{Q}_{fmkr}$ = upper bound for range $r$ of transportation flow of product family $f$ from warehouse $m$ to distribution center $k$

$\bar{Q}_{fklr}$ = upper bound for range $r$ of transportation flow of product family $f$ from distribution center $k$ to customer $l$

$Q_{ijm}$ = rate of flow of product $i$ transferred from plant $j$ to warehouse $m$

$Q_{imk}$ = rate of flow of product $i$ transferred from warehouse $m$ to distribution center $k$

$Q_{ikl}$ = rate of flow of product $i$ transferred from distribution center $k$ to customer zone $l$

$Q_{fjmr}$ = rate of flow of material of family $f$ from plant $j$ to warehouse $m$ in $r$th transportation flow intervals

$Q_{fmkr}$ = rate of flow of material of family $f$ from warehouse $m$ to distribution center $k$ in $r$th transportation flow intervals

$Q_{fklr}$ = rate of flow of material of family $f$ from distribution center $k$ to customer $l$ in $r$th transportation flow intervals

$R_{je}$ = total rate of provision of resource $e$ at plant $j$

$r$ = range of transportation flows with constant transportation cost

$s$ = product demand scenarios (superscript used in section 4 only)

$W_m^{\max}$ = maximum warehouse capacity

$W_m^{\min}$ = minimum warehouse capacity

$W_m$ = capacity of warehouse $m$

$X_{mk}$ = 1, if warehouse $m$ is assigned to distribution center $k$; 0, otherwise

$X_{kl}$ = 1, if distribution center $k$ is assigned to customer zone $l$; 0, otherwise

$Y_m$ = 1, if warehouse $m$ is to be established; 0, otherwise

$Y_k$ = 1, if distribution center $k$ is to be established; 0, otherwise

$Z_{fjmr}$ = 1, if the rate of flow of material in family $f$ from plant $j$ to warehouse $m$ is between $\bar{Q}_{fjm,r-1}$ and $\bar{Q}_{fjmr}$; 0, otherwise

$Z_{fmkr}$ = 1, if the rate of flow of material in family $f$ from warehouse $m$ to distribution center $k$ is between $\bar{Q}_{fmk,r-1}$ and $\bar{Q}_{fmkr}$; 0, otherwise

$Z_{fklr}$ = 1, if the rate of flow of material in family $f$ distribution center $k$ to customer $l$ is between $\bar{Q}_{fkl,r-1}$ and $\bar{Q}_{fklr}$; 0, otherwise

$\alpha_{im}$ = coefficient relating the capacity of warehouse $m$ to flow of product $i$ handled

$\beta_{ik}$ = coefficient relating the capacity of distribution center $k$ to flow of product $i$ handled

$\rho_{ije}$ = coefficient for resource $e$ used in plant $j$ to produce product $i$

$\psi_s$ = probability of product demand scenario $s$ occurring during the lifetime of the network

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