Homework 8

1) Consider a particle in a fluid. It is well known that the falling velocity is a constant. However when the number of particles is large their interaction affect the velocity. One expression proposed is

$$v(\gamma) = v_0\left(1 - \frac{\gamma}{\gamma_m}\right)\left(1 - \frac{\gamma/\gamma_m}{3\alpha - 1}\right)$$

γ = concentration of particles (particles/unit volume)

v = velocity

γ_m = # of particles for a settled suspension

$v(\gamma_m) = 0$

α: adjustable parameter

v_0 = velocity for a single particle

a) Show that the following equation governs the sedimentation

$$\frac{\partial \gamma}{\partial t} + v_0\left[\left(1 - \frac{2\gamma}{\gamma_m}\right)\left(1 - \frac{\gamma/\gamma_m}{3\alpha - 1}\right) - \left(1 - \frac{\gamma}{\gamma_m}\right)\frac{\gamma/\gamma_m}{3\alpha - 1}\right]\frac{\partial \gamma}{\partial x} = 0$$

b) Describe the solutions and determine if there is a possibility of shocks.

2) Greenberg page 975 - problem 13

3) " " 977 " 17

4) " " 991 " 10

5) " " 1042 " 5

Show all the details.

6) Greenberg page 1085 problem 5

7) " " 1091 " 5

8) Solve using a transform

$$\nabla^2 u = u_{xx} + u_{yy} = 0 \qquad |x| < \infty, \; x > 0, \; 0 < y < a.$$

$u(x,0) = f(x)$

$u(x,a) = g(x)$

$u(0,y) = r(x)$