Homework 6

1) Solve by giving a 2-term asymptotic expansion

$$\varepsilon y'' + \sqrt{x}\, y' - y = 0$$

$$y(0) = 0$$

$$y(1) = e^2$$

2) Solve

$$\left.\begin{aligned} \frac{d^2\mu}{dx^2} &= \phi^2 \mu v \\ \frac{d^2 v}{dx^2} &= \phi^2 \psi \mu v \end{aligned}\right\} \quad \phi \ll 1$$

$$\mu(0) = 1 \qquad \mu'(1) = 0$$

$$v'(0) = 0 \qquad v(1) = 0$$

3) Solve the hanging cable problem to whatever extent you can.

$$\varepsilon^2 \phi'' + kz \cos\phi - \sin\phi = 0$$

$$\phi(0) = \phi(1) = 0$$

5) $$\ddot{y}+y+\varepsilon\left[-y'+\tfrac{1}{3}y'^3\right]=0 \qquad \varepsilon<<1$$
$$y(0)=a \quad y'(0)=0$$
has only one periodic solution for some value $a=a^*$. Find it using strained coordinate and give a 3-Term expansion.
Hint: Expand a as a power series of ε and compute coefficients to avoid secular terms

6) Let $r(u,v)=u e^{\gamma\{1-\frac{1}{v}\}}$ in the CSTR problem.
Solve for $\phi>>>1$ and for $\phi<<<1$

7) Let $x^3y''-y=0$
Find a solution of the form
$$y=x^{\mu}e^{S(x)}\{1+a_1x^{\alpha}+a_2x^{\beta}+a_3x^{\gamma}+\ldots$$
Attempt to obtain at least a_2 and a_3

8) Let $y''+x^{-3/2}y'-x^{-2}y=0$.
a) $x=\infty$ is an irregular singular point. Prove it.
b) If you try $y=e^{S(x)}$ Assuming $S''<<S'^2$ turns out not to be correct. Try other balancing $(S')^2\sim x^{-3/2}S'$ $(S')^2>>x^{-3/2}S'$ or $(S')^2<<x^{-3/2}S'$ until you find one that works. Show the solution is of the form
$$y=x^{c}\{1+\alpha x^{-1/2}+\cdots\} \quad \text{as } x\to\infty$$