

Homework 4 (Individual work)

1) Let

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

Due: _____

find eigenvalues and eigenvectors

2) Is

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

diagonalizable?

3) If $A^2 = A$ prove that each eigenvalue is 0 or 1, such matrix is called idempotent

4) If $A^m = 0$ for some m the matrix is called nilpotent. Show such matrix is undiagonalizable

5) Compute e^A

$$A = \begin{bmatrix} -1 & 1 \\ -6 & 4 \end{bmatrix}$$

6) let $A = \begin{bmatrix} -1/2 & 1 \\ -1 & 3/2 \end{bmatrix}$

compute $\log(I+A)$

7) let $P^{-1}AP = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ $P = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

a) Find A

b) Show that $A^{10} = \begin{bmatrix} 1 & -10 & 65 & 165 \\ 0 & 1 & -10 & -55 \\ 0 & 0 & 1 & -10 \\ 0 & 0 & 0 & 3^{10} \end{bmatrix}$

c) Find A^{100}

8) Solve
$$\begin{aligned} x_1' &= 2x_1 - 2x_2 + 3x_3 + t \\ x_2' &= x_1 + x_2 + x_3 + t^2 \\ x_3' &= x_1 + 3x_2 - x_3 \end{aligned}$$

$$2x_1(0) + x_2(1) + x_3(1) = 0$$

$$x_1(1) + x_2(0) + 3x_3(0) = 1$$

9) Find the Green function of $(1-x)u'' + u' = f(x)$ $u(0) = u(1)$ $u'(1) = u(1)$ | Solve for $f(x) = \frac{1}{\sin(1+x)}$