Homework 3 (Due 9-11) (1)

(Individual)

1) Consider the Hermite equation

$$u'' - 2xu' + 2nu = 0$$

a) Obtain a series solutions for n integer, and n noninteger. and show in which case you obtain polynomials

b) Show the solutions are orthogonal, that is:

$$\int_{-\infty}^{\infty} e^{-x^2} H_m(x) H_n(x)\, dx = \delta_{mn}$$

Hint: $e^{-x^2/2} H_n(x)$ is a solution of $u'' + (2n+1-x^2)u = 0$

2) Consider the Riccati equation

$$y' + ay^2 = bx^n$$

a) Show that a transformation $y = u'/(au)$ converts it into a linear equation and attempt a series solution.

b) Use a second transformation, $u = v e^{Ax^p}$ to find another ODE. Find

appropriate values of A and p (2)
so that the resulting equation
simplifies to

$$v'' + 2cx^{p-1}v' + c(p-1)x^{p-2}v = 0$$

c) Solve it using

$$v = a_0 + a_1 x^p + a_2 x^{2p} + \dots a_n x^{np} \dots$$

Explain why such a series is
convenient.

d) Explain why a second solution
obtained from the same
series but using $-c$ instead
of c is l.i.

3) Attempt a series solution of

$$(y')^2 = a + by + cy^2$$

and see how you can obtain
a solution looking like either

$$y_1 = \alpha_1 \sin(\beta_1 x + \gamma_1) + \delta_1$$

or $\quad y_2 = \alpha_2 \sinh(\beta_2 x + \gamma_2) + \delta_2$

when are you going to get y_1 and when y_2?