Adv. Eng. Math

Homework 1

a) Individual work
b) Use Einstein notation whenever possible

1) Show that

$$\varepsilon_{ijk}\,\varepsilon_{klm} = \delta_{il}\,\delta_{jm} - \delta_{im}\,\delta_{jl}$$

2) Prove that

$$\underline{a}\times(\underline{b}\times\underline{c}) = \beta\,\underline{b} + \gamma\,\underline{c} \qquad (\beta, \gamma \text{ scalars})$$

and use 1) to derive

$$\underline{a}\times(\underline{b}\times\underline{c}) = (\underline{a}\cdot\underline{c})\,\underline{b} - (\underline{a}\cdot\underline{b})\,\underline{c}$$

3) If

$$\varepsilon_{ijk}\,a_i b_j c_k = 0$$

are $\underline{a}$ $\underline{b}$ $\underline{c}$ coplanar?

4) Show that the two lines

$$\underline{x} = \underline{a} + \underline{l}\,s \qquad s \text{ scalar}$$

$$\underline{x} = \underline{b} + \underline{m}\,s \qquad \underline{l}, \underline{m} \text{ unit vectors}$$

will intersect if $\underline{a}\,(\underline{l}\times\underline{m}) = \underline{b}\,(\underline{l}\times\underline{m})$ and find the intersection point.

5) Prove that $\varepsilon_{ijk}\,a_k$ is a second order tensor if $\underline{a}$ is a vector.

6) Prove that the average value of the normal component of the ~~stress~~ stress on a surface element over all directions is $\frac{1}{3}\,\mathrm{Tr}\,T$

Hint: Stress on a surface with normal $\underline{n}$ $= \underline{t}(n) = n_i T_{ij}$

7) Challenge (?): Let $\varphi(x)$ be a potential function and $r^2 = x_i x_i$

Show that

$$\varphi + \frac{a}{r}\,\varphi\left(\frac{a^2 x}{r}\right) - \frac{2}{ar}\int_0^a \lambda\,\varphi\left(\frac{\lambda^2 x}{r^2}\right)d\lambda$$

is a potential function and its normal derivative on a sphere $r=a$ vanishes.

8) Use Green's Theorem to show that

$$\int_V \underline{\nabla}\times\underline{b}\,dV = \int_S (\underline{n}\times\underline{a})\,dS$$

Hint $a_i = \varepsilon_{ijk} b_j \implies a_{,i} = \varepsilon_{kij}\, b_{j,i}$

9) Let $V = V_1 + V_2$. Let S' separate V_1 from V_2. Show that the divergence theorem holds for any V^* in V when

1) $\underline{a}$ is continuous in V with derivatives continuous in V_1 and V_2

2) The normal derivative is continuous across S'.

10) Show that $f(\underline{x}, t) = 0$ is a surface of the same material particles if and only if

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x_i} v_i = 0$$