The Signal Extraction Problem Revisited:
Its Impact on a Model of Monetary Policy*

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Abstract

This paper develops a dynamic stochastic general equilibrium (DSGE) model with sticky prices where agents have imperfect information on the stance and direction of monetary policy. Agents respond by using Kalman filtering to unravel persistent and temporary monetary policy changes in order to form optimal forecasts of future policy actions. Our results show that a sticky price model with imperfect information can account for several key effects of an expansionary monetary policy shock: the delayed and gradual increase in inflation, the large and persistent increase in output, the fall in the nominal interest rate, and the modest rise in the real wage.

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1 Introduction

Most empirical studies indicate that inflation gradually responds after a monetary policy shock with the peak occurring several quarters later.\textsuperscript{1} While empirical studies consistently yield that result, most dynamic stochastic general equilibrium models (DSGE) are unable to replicate it. Nelson [1998] shows that even the introduction of price stickiness into most DSGE models is not beneficial in producing a substantial lag in the peak inflation response.\textsuperscript{2} The failure of DSGE models to account for this behavior also affects their ability to generate persistent output effects and countercyclical nominal interest rate movements, i.e. the liquidity effect, after a monetary policy shock. Specifically, Chari, Kehoe, and McGrattan [2000] show that the rapid speed of price adjustment in sticky price models prevents them from producing persistent real effects. Kimball [1995] and King and Watson [1996], on the other hand, find that the large increases in expected inflation produced after a monetary disturbance prevent most sticky price models from generating the liquidity effect.\textsuperscript{3}

Given those results, there is a growing interest in developing a DSGE model that accounts for the empirical behavior of inflation over the business cycle. Christiano, Eichenbaum, and Evans [2001], Dotsey and King [2001], and others argue that reasonable modifications to DSGE models, such as adding variable capital utilization, help slow price adjustment and generate more output persistence by making marginal cost less elastic to changes in output. While those modifications are successful in generating additional output persistence, they are less effective in accounting for the gradual and delayed response in inflation.\textsuperscript{4} Alternatively, Mankiw and Reis [2002] using a partial equilibrium specification find that a sticky information model, with price setting behavior similar to Fischer’s [1977] contracting model, is better able to capture the dynamics of inflation than a standard sticky price model. Keen [2003], however, shows that Mankiw and Reis’ [2002] result is not sustainable when a Fischer [1977] style price setting rule is specified in a DSGE model.

One common feature of most business cycle models is the assumption that market participants have perfect information on the implementation and interpretation of monetary policy. Orphanides [2001], however, notes that the monetary authority experiences informational problems if it sets its policy instrument using variables such as output, prices, and potential output which are not accurately known until much

\textsuperscript{1}See Leeper, Sims, and Zha [1996] and Christiano, Eichenbaum, and Evans [1999] for a detailed discussion of this issue.

\textsuperscript{2}The only sticky price model examined by Nelson [1998] that produces a delayed and gradual response in inflation is Fuhrer and Moore’s [1995] inflation persistence model, but that model is not based on standard microfoundations of price determination.

\textsuperscript{3}Keen [2001] finds that the addition of a limited participation constraint to a sticky price model helps produce the liquidity effect for some but not all reasonable parameter values.

\textsuperscript{4}In Christiano, Eichenbaum, and Evans [2001], inflation peaks three years after a monetary policy shock, but for the first year and a half after the shock, inflation falls which is inconsistent with empirical observations.
later. Thus, monetary policy actions that are intended to be systematic responses to current economic conditions in hindsight may unintentionally shock the economy. We believe that the process underlying the monetary authority’s response after an unintentional or “nonpolicy” shock is different from when the monetary authority intentionally initiates a “policy” shock. That is, nonpolicy shocks are more transitory while policy shocks are more persistent. If our assumption that private agents cannot clearly distinguish between policy and nonpolicy shocks holds, then agents must form expectations of future monetary policy actions by solving a signal extraction problem.

Lucas’ [1975] “islands” model was one of the first to introduce imperfect information into a monetary model. In that model, firms can observe their own prices but cannot distinguish between changes in the average price level and changes in relative prices. Kydland and Prescott [1982] and Cooley and Hansen [1995] develop models that attempt to capture the spirit of Lucas’ [1975] model by assuming that market participants imperfectly observe changes in technology. In those models, technology shocks and not monetary policy shocks lead to business cycle movements. In more recent work, Woodford [2002] and Sims [2003] assume that monetary policy is completely transparent, but that agents have a limited capacity to process that information. Erceg and Levin [2003], on the other hand, presuppose that agents learn slowly about shifts in the target inflation rate. Our extension of the methodology of Lucas [1975] closely follows the approach of Amano, Hendry, and Zhang [1999]. In that paper, market participants imperfectly observe the stance of monetary policy and as a result, must solve a signal extraction problem to discern the monetary authority’s intentions based on their observations of the money growth rate. We integrate that style of imperfect information into a sticky price model, whereas Amano, Hendry, and Zhang [1999] integrate imperfect information into a limited participation model, which, at best, generates a one-period lag in the peak inflation response after a monetary policy shock.

This paper develops a DSGE model with sticky prices where agents have imperfect information on the objectives of the central bank. Specifically, agents can observe changes in the money growth rate but cannot observe the process underlying those changes. Agents respond by using the Kalman filter to form optimal forecasts of future changes in the money growth rate. When we make that assumption, our results show that a sticky price model can generate a gradual and delayed response in inflation, a persistent change in output, a countercyclical movement in the nominal interest rate, and a modest procyclical change in the real wage after a monetary policy shock. Those findings suggest that economists should consider integrating imperfect information into future models of the monetary transmission mechanism.

The remainder of the paper is structured as follows. Section 2 outlines our DSGE model with sticky prices. Section 3 describes the parameterization of the model. Section 4 presents our results for models with both perfect and imperfect information about the actions of the monetary authority. Section 5 concludes.
2 The Model

The DSGE model with sticky prices utilized in this paper assumes that market participants have imperfect information on the stance of monetary policy. Specifically, the monetary authority can change the money growth rate for either policy or nonpolicy related reasons. Policy shocks are fairly persistent money growth rate changes that are designed to capture exogenous shifts in monetary policy. Nonpolicy shocks, on the other hand, are temporary changes in the money supply. Those shocks capture the random errors resulting from the monetary authority’s systematic response to economic variables that are not accurately known at the time policy decisions are made. That specification allows us to examine the impact of monetary policy actions when agents do not have perfect information on the stance of monetary policy, but instead must form beliefs about that policy.

The basic structure of the model includes households, the capital supplier, monopolistically competitive firms, banks, and the monetary authority. After observing the money growth rate in the beginning of the period, households use their existing money balances to prepay firms for their consumption purchases and to make deposits at the banks. Banks lend those funds and the lump-sum transfers from the monetary authority to the capital supplier. The capital supplier, who is the owner of the capital in the economy, uses those funds to prepay firms for their investment purchases. Once production is complete, firms pay wages to the households and rent to the capital supplier, the capital supplier repays its loans with interest, and the banks return the deposits to the households with interest. Finally, households receive dividend payments from the firms, the capital supplier, and the banks.

2.1 The Monetary Authority

Monetary policy is conducted by lump-sum transfers to the banking sector, $M_t - M_{t-1}$, according to a money growth rule. Like Amano, Hendry, and Zhang [1999], monetary policy changes are characterized as either policy shocks or nonpolicy shocks. Policy shocks represent intentional changes to the monetary policy rule that are reasonably persistent. Such shifts cause a temporary change in the money growth rate but have a permanent effect on the money supply. Nonpolicy shocks symbolize unintentional errors resulting from the monetary authority’s systematic response to economic variables whose values are not accurately known at that time. We assume that the monetary authority reverses those nonpolicy shocks once they observe the actual values of the variables that they are responding to. Those shifts then have temporary but not permanent effects on both the money growth rate and the money supply. Consequently, the money supply growth rule comprises a policy component, $x_t^p$, and a nonpolicy component, $x_t^{np}$, such that:

$$\Delta \ln(M_t) = \ln(x_t^p) + \ln(x_t^{np}).$$

(1)
The policy component of (1) is a first order autoregressive [AR(1)] process as follows:
\[
\ln(x_p^t) = \rho \ln(x_p^{t-1}) + (1 - \rho) \ln(\mu) + \varepsilon_p^t,
\]
where \(\mu\) is the steady state gross money growth rate, \(1 > \rho > 0\), and \(\varepsilon_p^t \sim N(0, \sigma_p^2)\). In contrast, the nonpolicy component of (1) is a negative moving average [MA(N)] process, so that any changes in money growth are subsequently reversed. The nonpolicy component is given by:
\[
\ln(x_{np}^t) = \varepsilon_{np}^t + \sum_{i=1}^{N} b_i \varepsilon_{t-i}^{np},
\]
where \(\sum_{i=1}^{N} b_i = -1\) and \(\varepsilon_{np}^t \sim N(0, \sigma_{np}^2)\). Furthermore, \(\varepsilon_p^t\) and \(\varepsilon_{np}^t\) are orthogonal by assumption.

Most business cycle models assume that private agents have perfect information about the intentions of the monetary authority. Although the monetary authority clearly signals its intentions in some circumstances, there are many other situations where the objectives of the monetary authority are less clear. This paper seeks to analyze the effects of the latter situation. To conduct that analysis, we use techniques similar to those proposed by Kydland and Prescott [1982]. When private agents have imperfect information about the intentions of the monetary authority, they are unable to observe \(x_p^t\) and \(x_{np}^t\). Instead, private agents form expectations of future money growth based on their observations of the current money growth rate and their knowledge of the driving process of monetary policy shocks (\(\rho, \mu, b_i, \sigma_p^2,\) and \(\sigma_{np}^2\)). Given that information, we can use the Kalman filter to easily derive private agents’ expectations of future money growth.\(^5\)

To use the Kalman filter, the monetary policy rules in (1), (2), and (3) can be converted into the following state space representation:
\[
x_t = H' \cdot \xi_t
\]
\[
\xi_t = F \cdot \xi_{t-1} + \varepsilon_t,
\]
where \(x_t\) equals the observed variable, \(\Delta \ln(M_t)\), \(\xi_t\) comprises the unobservable variables such that:
\[
\xi_t = [\ln(x_p^t), \varepsilon_{np}^t, \varepsilon_{t-1}^{np}, \ldots, \varepsilon_{t-N}^{np}]',
\]
and \(\varepsilon_t\) is a \([N+2] \times 1\) vector of errors such that:
\[
\varepsilon_t = [\varepsilon_p^t, \varepsilon_{np}^t, 0, \ldots, 0]',
\]
Furthermore, \(H\) and \(F\) are \([N+2] \times 1\) and \(([N+2] \times [N+2])\) vectors of parameters, respectively. The Kalman filter then is employed on the state space system in (4) and (5) to form expectations of future money growth (i.e, \(E_t[\Delta \ln(M_{t+i})]\) for \(i = 1, 2, \ldots, \infty\)).\(^6\)

\(^5\)This model with imperfect information is structurally similar to that of Brunner, Cukierman, and Meltzer [1980].

2.2 Households

Each period, households begin with an initial level of money balances, $M_{t-1}$. Those balances are allocated between funding for consumption purchases, $S_t$, and deposits at the banks, $M_{t-1} - S_t$. The allocation of $M_{t-1}$ is influenced by a commodity market restriction that forces households to prepay for their consumption purchases. That restriction leads to the following cash constraint on consumption:

$$P_tc_t = S_t,$$  \hspace{1cm} (6)

where $c_t$ denotes consumption and $P_t$ denotes the price level. Households also have a fixed allotment of time each period, which is normalized to one, to divide between leisure, $l_t$, and labor, $n_t$:

$$n_t + l_t = 1.$$  \hspace{1cm} (7)

At the end of each period, households are paid by firms for their labor, $W_t n_t$, and receive their bank deposits with interest, $R_t (M_{t-1} - S_t)$, where $W_t$ denotes the nominal wage, and $R_t$ denotes the gross nominal interest rate. In addition, households receive dividend payments from firms, $D^f_t$, the capital supplier, $D^c_t$, and banks, $D^b_t$. Since all of that income is received after households have made current period consumption purchases, the income is stored as money according to the following equation:

$$M_t = W_t n_t + D^f_t + D^c_t + D^b_t + R_t(M_{t-1} - S_t).$$  \hspace{1cm} (8)

Next period, households will allocate those money balances between consumption purchases and savings.

Formally, households are infinitely-lived agents who have preferences for consumption and leisure. Each period, households choose $c_t$, $l_t$, $n_t$, $M_t$, and $S_t$ that maximizes the following expected utility function:

$$U_0 = E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \ln(c_t) + \theta \frac{l_t^{1-\zeta} - 1}{1 - \zeta} \right) \right]$$  \hspace{1cm} (9)

subject to the cash constraint, (6), the time constraint, (7), and the money accumulation equation, (8). The discount factor, $\beta$, is between 0 and 1 while $E_0$ is the expectational operator at time 0. Finally, the preference parameters $\zeta$ and $\theta$ are positive.

2.3 Capital Supplier

The capital supplier owns all of the capital in the model. Each period, the capital supplier rents capital to the firms, but it receives its rental payments, $P_t q_t k_t$, after production is complete. The capital supplier, however, must prepay firms for their investment purchases. As a result, the capital supplier must finance its investment purchases, $P_t i_t$, with loans from the banks. When the capital supplier receives its
rental payments at the end of the period, it uses those funds to pay off its loans with interest. Thus, the total cost of investment for the capital supplier is $R_t P_t i_t$.

The capital supplier maximizes the present value of its dividend stream to its owners, the households, by choosing the optimal amount of investment, $i_t$, and capital, $k_{t+1}$. Formally, the capital supplier seeks to maximize

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \lambda_t [P_t q_t k_t - P_t R_t i_t] \right],$$

where

$$\lambda_t = u_c(c_t, l_t)/(R_t P_t)$$

subject to the following capital accumulation equation:

$$k_{t+1} - k_t = \varphi(i_t/k_t)k_t - \delta k_t.$$

The fact that households own the capital supplier causes expected profits in period $j$ to be multiplied by the households’ marginal utility value of an additional dollar of profits in that period, $\lambda_{t+j}$. $\varphi$ is a Hayashi [1982] style capital adjustment costs parameter, where $i_t - \varphi(i_t/k_t)k_t$ are the resources lost in the conversion of investment to capital. Furthermore, we assume that it is more costly to incorporate new capital at higher rates than lower rates, so that $\varphi' > 0$ and $\varphi'' < 0$. Kimball [1995], King and Watson [1996], and Kim [2000], among others, argue that incorporating capital adjustment costs in the investment process is important in explaining interest rate movements over the business cycle.

2.4 Firms

Firms are monopolistically competitive producers of differentiated goods. Specifically, each firm $z$ on the unit interval produces a differentiated product, $y(z)$. Following Blanchard and Kiyotaki [1987], $y_t$ is a Dixit and Stiglitz [1977] aggregate of many differentiated goods:

$$y_t = \int_0^1 y(z)^{\epsilon/(\epsilon-1)} dz = \frac{1}{\epsilon-1} y_t^{1/(\epsilon-1)} y_t, (10)$$

where $-\epsilon$ is the price elasticity of demand for $y_t(z)$. That aggregate output comprises consumption goods sold to households and investment goods sold to the capital supplier such that:

$$y_t = c_t + i_t.$$

Each differentiated good, $y_t(z)$, sells at a price $P_t(z)$. Cost minimization on the part of households and the capital supplier implies that the demand schedule for $y_t(z)$ is a decreasing function of its relative price:

$$y_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon} y_t, (11)$$
where $P_t$ is a nonlinear price aggregate index of a continuum of differentiated goods:

$$P_t = \left[ \int_0^1 P_t(\zeta)^{1-\epsilon} d\zeta \right]^{1/(1-\epsilon)}.$$ 

Given its demand schedule, (11), each firm must determine the cost minimizing combination of inputs to produce its product and the price the firm is going to charge for that product.

Each firm $z$ on the unit interval hires labor, $n_t(z)$, and rents capital, $k_t(z)$, in perfectly competitive markets to produce its product, $y_t(z)$, according to a Cobb-Douglas production function:

$$y_t(z) = Z_t(k_t(z))^\alpha (n_t(z))^{1-\alpha},$$

where $Z_t$ is an aggregate technology shock that follows the autoregressive process:

$$\ln(Z_t) = \rho Z \ln(Z_{t-1}) + \varepsilon_{Zt},$$

where $1 > \rho_Z > 0$ and $\varepsilon_{Zt} \sim N(0, \sigma_Z^2)$.

Each period, firms hire labor from the households and rent capital from the capital supplier at the prevailing real wage rate, $W_t/P_t$, and capital rental rate, $q_t$, respectively. Given those input prices, each firm seeks to minimize its production costs:

$$(W_t/P_t)n_t(z) + q_t k_t(z)$$

subject to (12). The optimal labor and capital demands for the $z$th firm then are characterized as follows:

$$\psi_t (1 - \alpha) [k_t(z)/n_t(z)]^\alpha = W_t/P_t,$$

$$\psi_t \alpha [n_t(z)/k_t(z)]^{1-\alpha} = q_t,$$

where $\psi_t$ is the Lagrange multiplier on (12) and is interpreted as the real marginal cost.

Firms set their prices based on a Calvo [1983] style pricing rule. In each period, the probability that a firm receives an opportunity to adjust its price is $\eta$, while the probability that it must satisfy all demand at its previously established price is $(1 - \eta)$. Those firms able to adjust set a price, $P_t(z)$, that maximizes the present value of expected future profits to the households given the probability of future adjustment opportunities

$$\max_{P_t(z)} E_t \left[ \sum_{j=0}^{\infty} \beta^j (1 - \eta)^j \lambda_{t+j} [P_t(z)y_{t+j}(z) - W_{t+j}n_{t+j}(z) - P_{t+j}q_{t+j}K_{t+j}(z)] \right]$$

subject to the firm’s demand schedule, (11), and input factor demands, (13) and (14). The weight, $(1 - \eta)^j$, in (15) indicates the probability that the price remains fixed for
$j + 1$ periods. The maximization of (15) yields the following optimal price, $P_{0,t}^*$, for a price adjusting firm:

$$P_{0,t}^* = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{j=0}^{\infty} \beta^j (1 - \eta)^j E_t[\lambda_{t+j} \psi_{t+j} P_{t+j}^1]}{\sum_{j=0}^{\infty} \beta^j (1 - \eta)^j E_t[\lambda_{t+j} P_{t+j}^1]}.$$ 

In contrast, those firms that do not have an adjustment opportunity must charge the price, $P_{j,t}^*$, that they last changed $j$ periods ago.

### 2.5 Banks

Banks facilitate lending between the households and the capital supplier. At the beginning of the period, they receive deposits, $M_{t-1} - S_t$, from the households and a cash injection from the monetary authority, $X_t = M_t - M_{t-1}$. The capital supplier then borrows those funds from the banks to finance their investment purchases, $P_{i,t}$. The following equation summarizes that process:

$$P_{i,t} = M_{t-1} - S_t + X_t.$$ 

At the end of the period, the capital supplier repays its loans with interest, $R_t P_{i,t}$, and the households receive their deposits with interest, $R_t (M_{t-1} - S_t)$, and dividends, $R_t X_t$, from the banks.

### 3 Parameterizing the Model

The efficiency conditions and identity equations from each sector of the economy form a system of equations describing the model’s equilibrium. To induce stationarity, the distinct positive trend in the nominal variables $W_t$, $S_t$, $M_t$, $P_t(z)$, $X_t$, $D^p_t$, $D^f_t$, and $D^f_t$ caused by the constant increase in the money supply is eliminated by dividing those variables by $P_t$. That stationary-inducing transformation allows us to solve the model for its steady state. The system of equations then is linearized around its steady state. Finally, the model is solved by using the methodology of King and Watson [1998] combined with an extension of the methodology of King and Watson [1995] employed by Amano, Hendry, and Zhang [1999].

The parameter values specified in the model are consistent with those found in the literature. We assume that the discount factor, $\beta$, is 0.99, and preference parameter $\theta$ is selected so that the nonstochastic steady state value of labor, $\pi$, is 0.2. The preference specification in (9) gives rise to a labor supply elasticity that equals $4/\zeta$. Parameter estimates in Christiano and Eichenbaum [1992] suggest that the labor supply elasticity is at least 5.0, so $\zeta$ is set to 0.8. Capital’s share of output, $\alpha$, is set equal to 0.33 and the depreciation rate, $\delta$, is 0.025. The elasticity of demand, $\epsilon$, is set equal to 11, which means that the average mark-up in the economy is a modest 10%. The conditional probability of an adjustment, $\eta$, is set to 0.25, so that each
period a firm has a 25% chance of adjusting its price which implies that its price remains fixed for an average of four periods (quarters). The AR(1) coefficient, $\rho_Z$, for our technology law of motion is set to 0.95, while the variance of its innovations, $\sigma_Z^2$, is set equal to $(0.007)^2$. Since the solution method to our model is a linear approximation around the nonstochastic steady state, we do not need to specify a specific functional form for $\varphi(i_t/k_t)$. Instead, the average and marginal adjustment costs local to the steady state are assumed to be zero (i.e., $\varphi = i/k$ and $\varphi' = 1$), while elasticity of the investment-capital ratio to Tobin’s q, $\chi = [-{(i/k)\varphi''/\varphi'}]^{-1}$, is set to 1.8

The steady state gross money growth rate, $\mu$, is set to 1.01 which is consistent with a 4% annual inflation rate. Christiano, Eichenbaum, and Evans [1998] conclude that the exogenous M2 money growth rule is well approximated by an AR(1) specification with an autocorrelation coefficient of 0.5. As a result, we set the AR(1) coefficient, $\rho$, in our policy component equal to 0.5.9 Following Amano, Hendry, and Zhang [1999], we assume that the nonpolicy component follows a MA(1) process with a MA coefficient, $b_1$, equal to $-1$. That parameterization implies that the monetary authority completely reverses a nonpolicy shock in the subsequent period.

The parameterization of $\sigma_p^2$ and $\sigma_{np}^2$ is determined by setting a value for the money growth rate variance (i.e., $\text{var}(\ln(\Delta M_t))$) and by making an assumption about the amount of the money growth rate variance that is attributable to the policy component (i.e., $a = \text{var}(\ln(x_t^{p})) / \text{var}(\ln(\Delta M_t)))$.10 We set the variance of the money growth rate equal to $(0.00575)^2$, which is consistent with Nelson’s [1998] parameterization of Dow [1995]. Again, following Amano, Hendry, and Zhang [1999], we assume that the policy component accounts for only 10% of the variance in money growth. That low value for $a$ can be interpreted to mean that policy shocks are small, agents believe that most money growth shocks are nonpolicy in nature, or the monetary authority has a high level of credibility. Furthermore, specifying a value for $a < 1$ enables us to examine the impact of a monetary policy shock when the intentions of the central bank are unclear. The lower the value for $a$, the longer it takes agents to learn about a policy shock. There are many instances in which the monetary authority does not clearly indicate its intentions. Most monetary models, however, assume that agents clearly observe the stance of monetary policy. As a result, those models generate faster price adjustment and less output persistence after a policy shock. Our model, on the other hand, has the flexibility of examining the impact of a monetary policy shock both in the case where agents have perfect information on the intentions of the monetary authority ($a = 1$) and in the case where agents have imperfect information.

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7 Nelson [1998] uses these parameter values in his replication of Dow [1995].
8 An investment adjustment cost parameter $\chi = 1$ is consistent with Chirinko’s [1993] empirical examination of investment functions.
9 Amano, Hendry, and Zhang [1999] and Mankiw and Reis [2002] use the same parameter value in their models.
10 See appendix for details.
on the intentions of the monetary authority \((a < 1)\).

4 Results

This paper analyzes the ability of a DSGE model with sticky prices and imperfect information on the monetary authority’s intentions to account for key business cycle facts. Empirical studies suggest that an expansionary monetary policy shock produces a large and persistent increase in output, a gradual rise in the inflation rate over several periods, a decline in the nominal interest rate, and a modest boost in the real wage.\(^{11}\) Hence, any plausible model of the monetary transmission mechanism should account for that behavior.

This section initially analyzes the successes and failures of a sticky price model in generating key U.S. business cycle facts when agents are assumed to have perfect information on the intentions of the monetary authority. Next, we examine the impact of a nonpolicy shock on our model under an assumption of perfect information. The effects of the nonpolicy shock are presented because they provide valuable insight into the model’s behavior when agents have imperfect information on monetary policy. The model’s impulse responses to a policy shock then are examined under the assumption of imperfect information. After that, we assess the ability of our perfect and imperfect information specifications to capture the Phillips curve and inflation persistence properties observed in the data. Finally, the impact of the monetary authority’s credibility on the impulse responses to a policy shock is analyzed.

4.1 Perfect Information: Policy Shock

We begin by examining the effect of a 1\% expansionary policy shock on our sticky price model when agents have perfect information on the intentions of the monetary authority. The responses of output, inflation, the nominal interest rate, and the real wage to a policy shock are illustrated in Figure 1. That shock initially pushes up the money supply faster than prices can rise, which causes output to increase. As a result, labor demand rises which drives up the real wage. The policy shock, however, is observed fully in subsequent periods so that the output expansion is short-lived. Agents’ perfect knowledge of the future path of monetary policy also leads to a surge in both inflation and expected inflation. Consequently, inflation peaks on impact and not with a lag as empirically observed. The increase in inflation expectations also dominates any decline in the real interest rate, which causes the nominal interest rate to rise counterfactually. As a result, a sticky price model with perfect information is able to generate the output and real wage increases after a positive policy shock but cannot produce a lagged inflation peak or a decline in the nominal interest rate.

\(^{11}\)See Leeper, Sims, and Zha [1996] and Christiano, Eichenbaum, and Evans [1999] for examinations of this issue.
4.2 Perfect Information: Nonpolicy Shock

We now analyze the impact of a 1% positive nonpolicy shock on our sticky price model when agents have perfect information that the shock will be completely reversed in the following period. The impulse responses of output, the inflation rate, the nominal interest rate, and the real wage to a nonpolicy shock are shown in Figure 2. In the period when the nonpolicy shock occurs, price adjusting firms increase their prices only slightly because they realize that the money supply will return to its pre-shock level next period. Firms simply produce more output for one period. That higher output increases labor demand, which forces up the real wage. The fact that the shock is temporary causes households barely to adjust their level of current consumption. Therefore, the supply of resources available for investment rises, which in turn causes the nominal and real interest rates to fall.

When the nonpolicy shock is reversed in the following period, output, the nominal interest rate, and the real wage return to their pre-shock values. The inflation rate also falls but that drop is not a mirror image of its previous rise. That slow price level decline occurs because the next price adjustment opportunity for those firms who adjusted in the previous period is spread out over the next several periods. While the impact of a nonpolicy shock is short-lived, they have some important implications for the effects of a policy shock on our sticky price model with imperfect information.

4.3 Imperfect Information: Policy Shock

We next examine the impact of a 1% expansionary policy shock on our sticky price model when agents have imperfect information about the objectives of the monetary authority. Figure 3 reports the impulse responses for the imperfect information specification and compares them with the responses from the perfect information specification. In our model, $a$, the ratio of variance of the policy shock to variance of the money growth rate, is set to 0.1 in the imperfect information specification while $a$ equals 1.0 in the perfect information specification.\(^\text{12}\)

The impulse responses reported in Figure 3 illustrates that a sticky price model generates several important U.S. business cycle facts when agents have imperfect information about the persistence of a monetary disturbance. A key factor influencing the behavior of the imperfect information specification is that agents initially place a large weight on the probability that the monetary disturbance will be reversed in the following period. Therefore, immediately after the policy shock, the price adjusting firms increase their prices only modestly in comparison to the perfect information specification. As firms begin to learn that the monetary disturbance is persistent, price adjusting firms increasingly set higher prices. Such behavior enables the inflation rate to peak in the third period following the policy shock ($t = 4$).

\(^{12}\)The perfect information specification assumes agents observe that the monetary disturbance is a policy shock. This type of model is equivalent to an imperfect information specification where all of the variance in money growth is attributable to the policy shock.
The slow inflation response after a monetary disturbance brings about a larger and more persistent increase in output in the imperfect information specification than in the perfect information specification. Agents’ expectations of a reversal of the monetary disturbance lead to a smaller increase in expected inflation, which puts downward pressure on the nominal interest rate and enables the imperfect information specification to generate the liquidity effect. As for the real wage, the higher output level causes a larger rise in labor demand, while the lower inflation expectations lead to a smaller decline in labor supply than in the perfect information specification. The combined effect is that the real wage rises less with imperfect information than with perfect information, but the increase is moderately more persistent. As a result, a sticky price model with imperfect information successfully accounts for the following empirical effects of an expansionary policy shock: a gradual and lagged response in inflation, a large and persistent increase in output, a decline in the nominal interest rate, and a modest rise in the real wage.

Our analysis of the impulse responses in Figures 1-3 contains several important results. One, a sticky price model with perfect information can produce a positive response in output and the real wage after an expansionary monetary policy shock. Two, the model is unable to generate the lagged peak in inflation or the decline in the nominal interest rate after such a shock. Three, results for the sticky price model dramatically change when we assume that agents have imperfect information about the stance of monetary policy. Specifically, the imperfect information specification is able to produce the appropriate output, inflation, nominal interest rate, and real wage effects after a monetary policy shock.

4.4 The Phillips Curve Relationship

The Phillips curve relationship between output and inflation has been documented widely over the past few decades. This empirical relationship says that the inflation rate, \( \pi_t \), is a function of the expected inflation rate, \( \pi^e_t \), and the deviation of output from its potential, \([ (y_t - y^*_t)/y^*_t ] \), such that

\[
\pi_t = \pi^e_t + \gamma \left[ \frac{y_t - y^*_t}{y^*_t} \right],
\]

where \( \gamma > 0 \). While inconsistent with the assumption of rational expectations, a common and simple approach to identifying \( \pi^e_t \) is to set it equal to the lagged inflation rate, \( \pi_{t-1} \). Supporting that assumption is the conclusion by Barsky [1987] and Ball [2000] that inflation movements have been close to a random walk in the post-World War II period. Under the assumption that \( \pi^e_t = \pi_{t-1} \), the Phillips curve in (16) can be expressed as a relationship between the change in the inflation rate, \( \pi_t - \pi_{t-1} \), and \([ (y_t - y^*_t)/y^*_t ] \):

\[
\pi_t - \pi_{t-1} = \gamma \left[ \frac{y_t - y^*_t}{y^*_t} \right].
\]

Using annual data from 1961-2002, Figure 4 shows that the Phillips relationship between \( \pi_t - \pi_{t-1} \) and \([ (y_t - y^*_t)/y^*_t ] \) generally has a positive slope as suggested by
Given the theoretical and empirical relationships outlined in (17) and in Figure 4, we calculate the correlation between output, $y_t$, and both the one- and two-year changes in the inflation rate, $\pi_{t+2} - \pi_{t-2}$ and $\pi_{t+4} - \pi_{t-4}$, and compare them with the results generated in our models with perfect and imperfect information. The data used to estimate those correlations covers the period 1959Q1-2003Q2. Output is the log of real GDP divided by the civilian, noninstitutional population, age 16 and over. The output data is detrended by passing it through Baxter and King’s [1999] approximate band-pass filter. As for the inflation rate, four different measures are used: GDP price deflator, CPI, core CPI, and PPI.

Table 1 shows the correlation between output and the change in the inflation rate in the data and in the perfect and imperfect information specification. Correlation coefficients for real per capita GDP and the GDP price deflator, CPI and core CPI range from 0.46 to 0.54 for the one-year change and 0.54 to 0.61 for the two-year change. Real output’s correlation with PPI is slightly smaller for both the one-year change, 0.27, and two-year change, 0.41, but is still strongly positive. The average correlation coefficient for the one-year change is 0.44 and for the two-year change is 0.54. The strength and uniformity of those results indicates that rising output is associated with higher inflation.

We next evaluate the ability of the models with perfect and imperfect information to generate the positive correlation between output and the change in the inflation rate. The perfect information specification produces a modest positive correlation of 0.18 for the one-year change in inflation and 0.25 for the two-year change in inflation. Those coefficients, however, are significantly less than their average values observed in the data. In the imperfect information specification, the correlation coefficient of 0.56 for the one-year change in inflation is slightly higher, but not statistically significant, than the data average of 0.44, whereas the coefficient of 0.54 for the two-year change in inflation is equal to its data average. Therefore, we conclude that a sticky price model with imperfect information is better able to capture the relationship between economic activity and the change in inflation than a sticky price model with perfect information.

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13 In Figure 8, the change in the inflation rate is measured using the CPI price index, output is GDP divided by the 16 plus, civilian, noninstitutional population, and potential output is calculated using the Congressional Budget Office’s measure of potential output divided by the 16 plus, civilian, noninstitutional population.

14 400 simulations of the model are calculated on a sample set of 154 periods which is conditioned on the -10th observation. Conditioning the simulations on more observations does not impact the results.

15 The band-pass filter is used with 12 symmetric leads and lags to remove any portions of output data not associated with cycles of 2-32 quarters.

16 We do not examine core PPI because that data series does not begin until 1974.

17 The qualitative results in Table 1 remain the same when the HP filter is employed as an alternative to Baxter and King’s [1999] approximate band-pass filter.
4.5 Inflation Persistence

Fuhrer and Moore [1995] demonstrate that the standard sticky price model with perfect information generates far too little inflation persistence. Specifically, inflation data exhibits a strong positive autocorrelation that decays very slowly, whereas sticky price models with perfect information generate positive inflation autocorrelations that are strong initially but diminish rapidly as the order of autocorrelation increases. The inability to produce a plausible degree of inflation persistence motivates us to compare the inflation autocorrelations observed in the data with those generated by our perfect and imperfect information models.

Table 2 shows the first five inflation autocorrelations ($\rho_\pi(k)$ for $k = 1, 2, ..., 5$) in both the data and in the perfect and imperfect information specifications. The first-order inflation autocorrelations, $\rho_\pi(1)$, for the GDP price deflator, CPI and core CPI range from 0.84 to 0.90. The higher order autocorrelations for those three inflation measures remain quite large as $k$ increases. For example, $\rho_\pi(5)$ ranges from 0.61 to 0.74. The autocorrelation coefficients for PPI are somewhat weaker than the other three inflation measures, but still show a high degree of autocorrelation ($\rho_\pi(1) = 0.62$ and $\rho_\pi(5) = 0.37$). Thus, we conclude that inflation displays substantial persistence.

The autocorrelation coefficients generated for the perfect and imperfect information specifications show that neither model can generate the degree of inflation persistence observed in the data. Both the perfect and imperfect information specifications produce strong first-order autocorrelations ($\rho_\pi(1)$ equals 0.76 and 0.73, respectively), which are consistent with the data average ($\rho_\pi(1) = 0.80$). The autocorrelation coefficients for both models, unlike in the data, quickly diminish in size as $k$ increases. At the third-order autocorrelation, both models produce coefficients ($\rho_\pi(3)$ equals 0.39 and 0.31, respectively) that are significantly less than the data average ($\rho_\pi(3) = 0.74$). The difference is more pronounced at the fifth-order autocorrelation where the coefficients for both models ($\rho_\pi(1)$ equals 0.17 and 0.09, respectively) are not significantly different from 0, although the data average of $\rho_\pi(5) = 0.57$ indicates that inflation is still very autocorrelated.

While our results seem to suggest that models with perfect and imperfect information cannot generate the inflation persistence observed in the data, Nelson [1998] notes that the number and type of random shocks in the model can have “striking effects” on their ability to produce inflation persistence. In fact, Ireland [2001] succeeds in generating considerable inflation persistence in a sticky price model with perfect information by including stochastic shocks to aggregate demand and money demand. Since the autocorrelation coefficients for our sticky price model with imperfect information mirrors its respective values with perfect information, Ireland’s [2001] results suggest that generating inflation persistence in a sticky price model with imperfect

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18 All of the inflation data series in Table 2 are detrended using Baxter and King’s [1999] approximate band-pass filter. This detrending method is slightly different from Table 1 where differencing eliminates the trend from the inflation data.
information can be achieved by including those stochastic shocks in our model. That task, however, remains for future research. Finally, Mankiw and Reis [2002] place our results in perspective when they state that “the key empirical fact that is hard to match, however, is not the high autocorrelations of inflation, but the delayed response of inflation to monetary policy shocks.”

4.6 Credibility and the Impact of a Policy Shock

Goodfriend [1993] notes that firms have more pressure to raise their prices during periods of expansionary monetary policy when the monetary authority’s credibility for its commitment to maintain a low and stable inflation rate is in question. Hence, a plausible model of the monetary transmission mechanism should produce a more gradual adjustment in inflation after a monetary policy shock when the monetary authority has a large degree of credibility than when it has little credibility. That slow adjustment in prices and the resulting moderation in inflation and expected inflation also leads to additional output persistence and a more pronounced decline in the nominal interest rate. This section examines the impact of credibility on the responses of output, inflation, the nominal interest rate, and the real wage to a policy shock in our model with imperfect information.

In our model, the measure of credibility is the parameter $a$, which is the percentage of the money growth rate variance that is attributable to the policy component of the monetary policy rule. When $a$ is small and close to zero, most of the variation in money growth comes from the nonpolicy component, so it is expected that nearly all of the money growth shocks will be reversed in the future. Thus, the underlying inflation rate is not expected to change, which is consistent with an environment where the monetary authority has a significant amount of credibility. In the case where $a$ is large and close to one, the monetary authority has little credibility because most monetary policy shocks are expected to be permanent and accordingly, prices are expected to rise. Since parameter $a$ is our measure of credibility, this section can alternatively be interpreted as a sensitivity analysis of the impulse responses in our imperfect information specification to various parameterizations of $a$.

Figure 5 shows the responses of output, inflation, the nominal interest rate, and the real wage to a 1% expansionary policy shock in the model with imperfect information. The impulse response for each variable is examined for four different degrees of credibility, $a = 0.50$, $a = 0.30$, $a = 0.10$, and $a = 0.01$, where $a = 0.10$ is the parameterization used in our benchmark model. Consistent with the arguments made in Goodfriend [1993], inflation’s initial response is strongest when the monetary authority lacks credibility and is weakest when the monetary authority has substantial credibility. Specifically, the inflation rate peaks at a lag of three periods ($t = 4$) in the benchmark parameterization ($a = 0.10$) but improves to a lag of five periods ($t = 6$) where $a = 0.01$. When the monetary authority has less credibility, our model’s ability to generate a lagged peak in the inflation rate slowly erodes, but is still present at a
lag of one period when credibility is as low as $a = 0.50$.

Credibility also has a considerable effect on the behavior of output, the nominal interest rate, and the real wage after a policy shock. As expected, the degree of output persistence increases and the decline in the nominal interest rate strengthens as the amount of credibility rises. In fact, the fall in the nominal interest rate is very short-lived when credibility is low ($a = 0.50$ and $a = 0.30$), but that decrease is much more persistent at moderate and high degrees of credibility ($a = 0.10$ and $a = 0.01$). Higher credibility also moderates the rise in the real wage and makes it more persistent. Specifically, the initial real wage increase is strongest when credibility is low ($a = 0.50$ and $a = 0.30$), but at six periods after the policy shock ($t = 7$) the real wage response is strongest when credibility is high ($a = 0.10$ and $a = 0.01$).

5 Conclusion

Most DSGE models of the monetary transmission mechanism assume that agents fully understand the nature of the monetary policy process. An analysis of post-war U.S. business cycle data, however, suggests that there are instances where the public has not clearly understood the intentions of the monetary authority. This paper examines the impact of a monetary policy shock when agents do not observe the persistence of a policy shock. Instead, agents learn about shifts in monetary policy slowly over time by monitoring changes in the money supply. To obtain their optimal forecasts of future money growth, agents use the Kalman filter to disentangle persistent and transitory changes in monetary policy. Those forecasts are instrumental in determining the responses of both real and nominal variables in our models.

The concept that agents have imperfect information about the stance and direction of monetary policy is a critical modification to a DSGE model with sticky prices because it diminishes the expectations that a monetary policy shock is persistent. Those reduced expectations assist in producing a large and persistent output effect, a gradual and lagged response in the inflation rate, a countercyclical response in the nominal interest rate, and a modest procyclical change in the real wage. Our findings in a model with imperfect information complement existing literature which emphasizes model modifications, such as variable capital utilization, that slow the speed of price adjustment by decreasing the elasticity of marginal cost to output.

The introduction of imperfect information into a DSGE model of monetary policy is a natural extension of the blending of New Keynesian mechanisms and classical business cycle models. While our assumption of imperfect information is not appropriate for every business cycle, it does provide an explanation for behavior that previously presented a theoretical puzzle for macroeconomists. Our modification to a standard DSGE model represents another step forward in the development of a usable structural model for policy analysis.
A Appendix

This appendix parameterizes values for \( \sigma_p^2 \) and \( \sigma_{np}^2 \) from the money growth process in a manner similar to Amano, Hendry, and Zhang [1999] and Andolfatto et al. [2000]. Recall from the text that the money growth process comprises an AR(1) policy component and a MA(N) non-policy component such that:

\[
\Delta \ln(M_t) = \ln(x_t^p) + \ln(x_t^{np}).
\]

The assumption that \( \ln(x_t^p) \) and \( \ln(x_t^{np}) \) are orthogonal implies

\[
Var(\Delta \ln(M_t)) = Var(\ln(x_t^p)) + Var(\ln(x_t^{np})). \tag{A1}
\]

A variable \( a \) is defined as the fraction of the variance of \( \Delta \ln(M_t) \) that is attributable to \( \ln(x_t^p) \), i.e.,

\[
a = \frac{Var(\ln(x_t^p))}{Var(\Delta \ln(M_t))}. \tag{A2}
\]

Given (A1) and (A2), the fraction of the variance of \( \Delta \ln(M_t) \) that is attributable to \( \ln(x_t^{np}) \) is

\[
(1 - a) = \frac{Var(\ln(x_t^{np}))}{Var(\Delta \ln(M_t))}. \tag{A3}
\]

We know that the variance for the AR(1) policy component is

\[
Var(\ln(x_t^p)) = \frac{\sigma_p^2}{(1 - \rho^2)}, \tag{A4}
\]

and the variance for the MA(N) non-policy component is

\[
Var(\ln(x_t^{np})) = \sigma_{np}^2 \left( 1 + \sum_{i=1}^{N} b_i^2 \right). \tag{A5}
\]

For convenience, we define \( Var(\Delta \ln(M_t)) = 1 \). As a result, we can substitute (A2) into (A4) and (A3) into (A5) to derive the following parameterized values for \( \sigma_p^2 \) and \( \sigma_{np}^2 \)

\[
\sigma_p^2 = a(1 - \rho^2),
\]

\[
\sigma_{np}^2 = \frac{(1 - a)}{\left( 1 + \sum_{i=1}^{N} b_i^2 \right)}. \]
References


Table 1: Correlation Between Output, \( y_t \), and the Change in the Inflation Rate, \( \pi_{t+j} - \pi_{t-j} \).
(standard deviation in parentheses)

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<tr>
<th>Data</th>
<th>( j = 2 )</th>
<th>( j = 4 )</th>
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</thead>
<tbody>
<tr>
<td>GDP Deflator</td>
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<tr>
<td>CPI</td>
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<td>0.54</td>
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<tr>
<td>Core CPI</td>
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<tr>
<td>PPI</td>
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<td>0.41</td>
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<td>Average</td>
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<table>
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<th>Models</th>
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<th>( j = 4 )</th>
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</thead>
<tbody>
<tr>
<td>Perfect Information (( a = 1.0 ))</td>
<td>0.18 (0.11)</td>
<td>0.25 (0.14)</td>
</tr>
<tr>
<td>Imperfect Information (( a = 0.1 ))</td>
<td>0.56 (0.07)</td>
<td>0.54 (0.10)</td>
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</table>
Table 2: Inflation Autocorrelations, $\rho_\pi(k)$
(standard deviation in parentheses)

<table>
<thead>
<tr>
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<th>4</th>
<th>5</th>
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<tr>
<td>GDP Price Deflator</td>
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<td>0.85</td>
<td>0.82</td>
<td>0.80</td>
<td>0.74</td>
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<tr>
<td>CPI</td>
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<td>0.76</td>
<td>0.77</td>
<td>0.68</td>
<td>0.61</td>
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<td>CoreCPI</td>
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<td>PPI</td>
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<td>0.51</td>
<td>0.60</td>
<td>0.46</td>
<td>0.37</td>
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<tr>
<td>Average</td>
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<td>0.73</td>
<td>0.74</td>
<td>0.66</td>
<td>0.59</td>
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</table>

<table>
<thead>
<tr>
<th>Models</th>
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<th>2</th>
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<tbody>
<tr>
<td>Perfect Information</td>
<td>0.76</td>
<td>0.56</td>
<td>0.39</td>
<td>0.27</td>
<td>0.17</td>
</tr>
<tr>
<td>$(a = 1.0)$</td>
<td>(0.05)</td>
<td>(0.09)</td>
<td>(0.11)</td>
<td>(0.13)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>Imperfect Information</td>
<td>0.73</td>
<td>0.49</td>
<td>0.31</td>
<td>0.18</td>
<td>0.09</td>
</tr>
<tr>
<td>$(a = 0.1)$</td>
<td>(0.05)</td>
<td>(0.09)</td>
<td>(0.11)</td>
<td>(0.12)</td>
<td>(0.13)</td>
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Figure 1: Impact of a Policy Shock on our Perfect Information Specification

Output

Inflation Rate

Nominal Interest Rate

Real Wage
Figure 2: Impact of a Nonpolicy Shock on our Perfect Information Specification
Figure 3: Impact of a Policy Shock on our Imperfect Information Specification

- **Output**
  - Imperfect Information
  - Perfect Information

- **Inflation Rate**
  - Imperfect Information
  - Perfect Information

- **Nominal Interest Rate**
  - Imperfect Information
  - Perfect Information

- **Real Wage**
  - Imperfect Information
  - Perfect Information
Figure 4: The Phillips Curve Relationship Between GDP and Inflation, 1961-2002
Figure 5: Credibility and the Impact of a Policy Shock

Output

- Dotted line: $a=0.50$
- Dashed line: $a=0.30$
- Solid line: $a=0.10$
- Dash-dotted line: $a=0.01$

Inflation Rate

Nominal Interest Rate

Real Wage

- Dotted line: $a=0.50$
- Dashed line: $a=0.30$
- Solid line: $a=0.10$
- Dash-dotted line: $a=0.01$